Gratitude

I am immensely grateful to the following person for the preparation and revision of this lecture.

- Mr. Md. Minal Nahin, Graduate Research Assistant, ME, IUPUI
- Mr. Anjan Goswami, Assistant Professor, MPE, AUST
- Dr. Md. Zahirul Haq, Professor, ME, BUET
- Dr. Sumon Saha, Assistant Professor, ME, BUET
## Tentative Course Plan

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1 with Mr. Abu Jafar Rasel
2 Each lecture is approximately 120 min.

---

### Marks Distribution

1. **Quizes 50%**
2. **Classwork 40%**
3. **Assignment 10%**
Quizes 50%
Classwork 40%
Assignment 10%
Classwork will be held at the beginning of the class.

There will be at least one sudden quiz.
Marks Distribution

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2. Classwork 40%
3. Assignment 10%

4. Classwork will be held at the beginning of the class.
5. There will be at least one sudden quiz.
6. Absence from both quizzes will result in a 'F' grade.

7. Calculators are not allowed for classwork and quiz.
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7. Calculators are not allowed for classwork and quiz.
8. Rough calculations on the question paper is not allowed either.
9. For rough calculations you will need to use MATLAB command window.
Control engineering or control systems engineering is the engineering discipline that applies control theory to design systems with desired behaviors.

The practice uses sensors to measure the output performance of the device being controlled and those measurements can be used to give feedback to the input actuators that can make corrections toward desired performance.

When a device is designed to perform without the need of human inputs for correction it is called automatic control (such as cruise control for regulating a car’s speed).

Multi-disciplinary in nature, control systems engineering activities focus on implementation of control systems mainly derived by mathematical modeling of systems of a diverse range.
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Multi-disciplinary in nature, control systems engineering activities focus on implementation of control systems mainly derived by mathematical modeling of systems of a diverse range.
**Liquid Level Control**

![Diagram of liquid level control](image)

**Automatic Control**

![Diagram of automatic control](image)
Watt’s Flyball Governor

Elements of Measurement System

Consists of 3 basic elements
The term **Control** means to regulate, to direct or to command.

A **control system** is defined as a combination of devices and components connected or related so as to command, direct or regulate itself or another system.

---

**Control System**

The term **Control** means to regulate, to direct or to command.

A **control system** is defined as a combination of devices and components connected or related so as to command, direct or regulate itself or another system.
Classification of Control Systems

Toaster

Air Conditioner

- **Open Loop System**: Does not correct for feedback/disturbance. Also called non-feedback system.

- **Close Loop System**: Corrects for feedback/disturbance. Also called feedback system.
Classification of Control Systems

- **Open Loop System**: Does not correct for feedback/disturbance. Also called non-feedback system.
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Block diagrams of control systems: a. open-loop system; b. closed-loop system
Definition and Modeling of Systems

- **System** can be thought of as a box which has an input and an output.
- **Equations** are used to describe the relationship between the input and output of a system.
- **Response** of a system is a measure of its fidelity to its purpose.
- **Modeling** is the process of representing the behavior of a system by a collection of mathematical equations & logics.
- **Simulation** is the process of solving the model and it is performed on a computer.
- In this course, we’re going to learn to simulate systems using **MATLAB-Simulink**.
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Introduction

- MATLAB developed by The MathWorks Inc., stands for Matrix Laboratory.
- It is primarily used to perform scientific computation and visualization.
- Run MATLAB

MATLAB Environment consists of

1. Command Window: To execute commands.
2. Command History: To keep track of previously entered commands
3. Workspace: Collection of all variables, their data type and size.
5. Figure Window: To display plots or graphics.
6. Edit Window: To create or modify a new or existing file respectively.

To restore default window go to Menu: Desktop/Desktop Layout/Default
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Basic Plotting (2D & 3D)

plot(x,y)
Plot the function \( y = x - 3 \) over the range \(-1 \leq x \leq 1\)

Solution:

- Create a vector of dependent variable \( x \) within the given range.
  \[ x = -1 : 0.1 : 1 \]
**Basic Plotting (2D & 3D)**

```latex
code
plot(x,y)
```

Plot the function \( y = x - 3 \) over the range \(-1 \leq x \leq 1\)

**Solution:**

- Create a vector of dependent variable \( x \) within the given range.
  \[ x = -1 : 0.1 : 1 \]

- Independent variable \( y \) is assigned within the given range. \[ y = x - 3 \]

- Plot the graph in a figure window by \[ \textit{plot}(x, y) \]
Basic Plotting (2D & 3D)

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- Plot the graph in a figure window by \( \text{plot}(x,y) \)

- Note the use of \( .(\text{dot}) : (\text{colon}) \) and \( ;(\text{semicolon}) \) in MATLAB

- The function \texttt{linspace} is used to assign 100 values within a given range.
**Basic Plotting (2D & 3D)**

**plot(x,y)**

Plot the function \( y = x - 3 \) over the range \(-1 \leq x \leq 1\)

**Solution:**

- Create a vector of dependent variable \( x \) within the given range.
  \( \rightarrow x = -1 : 0.1 : 1 \)
- Independent variable \( y \) is assigned within the given range. \( \rightarrow y = x - 3 \)
- Plot the graph in a figure window by \( \rightarrow plot(x,y) \)
- Note the use of \.(dot)\ : (colon) and ; (semicolon) in MATLAB
- The function \textit{linspace} is used to assign 100 values within a given range.
  \( \rightarrow x = \textit{linspace}(-1,1) \)

**Meshgrid & Surf**

Plot the function \( z = e^{-x^2-y^2} \) over the range \(-2 \leq x \leq 2 \) and \(-2 \leq y \leq 2\)
Basic Plotting (2D \& 3D)

Meshgrid \& Surf

Plot the function \( z = e^{-x^2-y^2} \) over the range \(-2 \leq x \leq 2\) and \(-2 \leq y \leq 2\)

Solution:

\[
\begin{align*}
&\text{>> } x= \text{linspace(-2,2);} \\
&\text{>> } y= \text{linspace(-2,2);} \\
&\text{>> } \{X,Y\} = \text{meshgrid}(x,y); \\
&\text{>> } Z= \exp(-X.^2 -Y.^2); \\
&\text{>> } \text{surf}(X,Y,Z)
\end{align*}
\]

Note the use of Plot tools
Basic Plotting (2D & 3D)

Meshgrid & Surf

Plot the function \( z = e^{-x^2-y^2} \) over the range \(-2 \leq x \leq 2\) and \(-2 \leq y \leq 2\)

Solution:

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>> \ [X,Y] = \text{meshgrid}(x,y);
>> \ Z = \text{exp}(-X.^2 - Y.^2);
>> \text{surf}(X,Y,Z)
\]

Note the use of \text{Plot tools}

Try \text{mesh(X,Y,Z)} instead of \text{surf(X,Y,Z)}

Practice

Plot the function \( c = a^2 - b^2 \) over the range \(-2 \leq a \leq 2\) and \(-2 \leq b \leq 2\)
Plot the function \( y = \sin x \) over the range \(-2\pi \leq x \leq 2\pi\)
Plot the function \( y = x^3 - x^2 + 3 \) over the range \(-4 \leq x \leq 4\)

Some useful functions for plotting

\text{figure} \Rightarrow \text{Creates an empty figure window.}
Some useful functions for plotting

- **figure** ⇒ Creates an empty figure window.
- **title('string')** ⇒ Creates title in figure window.
- **xlabel('x-axis')** ⇒ Creates x-axis label in figure window.
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1. `figure` ⇒ Creates an empty figure window.
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6. **shading interp** ⇒ Changes shading of plot (faceted, flat and interp).
7. **hold on** ⇒ holds the current figure window for further plotting.
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8. ezplot(‘x’) ⇒ plots the graph of the string input function

9. area(x,y) ⇒ same as plot(x,y) but the area under the curves are filled.
Some useful functions for plotting

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2. `title('string')` ⇒ Creates title in figure window.
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5. `legend('a,b')` ⇒ Creates legend in figure window.
6. `shading interp` ⇒ Changes shading of plot (faceted, flat and interp).
7. `hold on` ⇒ holds the current figure window for further plotting.
8. `ezplot('x')` ⇒ plots the graph of the string input function
9. `area(x,y)` ⇒ same as `plot(x,y)` but the area under the curves are filled.
10. `bar(x,y)` ⇒ plots bar graph.
11. `mesh(x,y,z)` ⇒ same as `surf(x,y,z)` except patches between lines are not filled.
Solving Linear Equations: *linsolve* function

Any linear system of equations must have 0, 1 or infinite number of solutions.

*linsolve*(A,B) takes coefficient matrix A and constant matrix B as input and solves the system.

Solve the following system.

\[
\begin{align*}
    x + 3y + z &= 5 \\
    2x - y + 2z &= 17 \\
    3x + 4y + 5z &= 26
\end{align*}
\]
Solving Linear Equations: `linsolve` function

Any linear system of equations must have 0, 1 or infinite number of solutions. `linsolve(A,B)` takes coefficient matrix A and constant matrix B as input and solves the system.

Solve the following system.

System of Linear equations

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\begin{align*}
x + 3y + z &= 5 \\
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3x + 4y + 5z &= 26
\end{align*}
\]

Solution visualization

Draw the plot of the following equations in same graph.

System of Linear equations with one solution

\[
\begin{align*}
x + 3y &= 5 \\
2x - y &= 17
\end{align*}
\]

Draw the mesh plot of the following equations in same graph.

System of Linear equations with infinite solutions

\[
\begin{align*}
x + 3y + z &= 5 \\
2x - y + 2z &= 17
\end{align*}
\]
Solution visualization (contd.)
MATLAB interprets an nth order polynomial as a row vector of n + 1

Consider a polynomial \( P(s) = s^4 + 3s^3 - 15s^2 - 2s + 9 \)

Enter it in MATLAB as row vector \( P \) as follows

\[ P = [1 \ 3 \ -15 \ -2 \ 9] \]

or \( P = [1, 3, -15, -2, 9] \)

To evaluate the value of \( P \) at \( s = 3 \) or \( P(3) \) use the following command \( \text{polyval}(P,s) \)

\( \text{polyval}(P,3) \)

MATLAB will show you the value of \( P(3) \) to be 30.

If there are missing terms, zeros must be entered at the appropriate places.
Polynomials

- **MATLAB** interprets an nth order polynomial as a row vector of \( n + 1 \)

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  P = [1 \ 3 \ -15 \ -2 \ 9] \leftarrow
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  \[
  \text{polyval}(P,3) \\
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Evaluate the value of

\[
y = 2s^2 + 3s + 4 \text{ at } s = 1, -3
\]
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\[ y = 2s^2 + 3s + 4 \]
at \( s = 1, -3 \)

Answer: \( y(1) = 9 \quad y(-3) = 13 \)

Evaluate the value of
\[ y = 2s^3 + 1 \]
at \( s = -4 \)
Practice

Evaluate the value of
\[ y = 2s^3 + 1 \] at \( s = -4 \)

Answer: \( y(-4) = -127 \)

Roots

- MATLAB can find the roots of a polynomial \( P \) by the command `roots(P)`
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- or $r = \text{roots}(P)$
- The answer is represented in a column vector.

Find the roots of $s^2 + 3s + 2$

Answer: $-2, -1$
If you know the roots(r) of a polynomial P, MATLAB can find the polynomial by the command `poly(r)`.

- Represent the roots by a column vector.
Find the Polynomial

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  - \( p = \text{poly}(r) \)
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- Represent the roots by a column vector.

  - `r = [-1; -2] ←`

  - `p = poly(r) ←`

- The answer is represented in a row vector `p = [1 3 2]`.

Find the polynomial whose roots are -5.5745, 2.5836, -0.7951, 0.7860.
Find the Polynomial

- If you know the roots\(r\) of a polynomial \(P\), MATLAB can find the polynomial by the command `poly(r)`.
- Represent the roots by a column vector.
  \[ r = [-1; -2] \]
  \[ p = poly(r) \]
- The answer is represented in a row vector \(p = [1 \ 3 \ 2]\).

Find the polynomial whose roots are -5.5745, 2.5836, -0.7951, 0.7860.

Answer: \(P(s) = s^4 + 3s^3 - 15s^2 - 2s + 9\)

Conv\((x,y)\) Function: Convolution

- MATLAB can multiply two polynomials by the command `conv(x,y)`.
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- Define the polynomials separately.
- The product is returned as a row matrix.
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- Define the polynomials separately.
- The product is returned as a row matrix.
- In case there is more than two polynomials perform multiplication two at a time.

Multiply (s + 2) and s^2 + 4s + 8
Conv(x,y) Function: Convolution

- MATLAB can multiply two polynomials by the command conv(x,y)
- Define the polynomials separately.
- The product is returned as a row matrix.
- In case there is more than two polynomials perform multiplication two at a time.

Multiply \((s + 2)\) and \(s^2 + 4s + 8\)

Answer: \(s^3 + 6s^2 + 16s + 16\)

Practice

Evaluate the product of \((s + 3)\), \((s + 6)\) and \((s + 2)\)
Evaluate the product of \((s + 3)\), \((s + 6)\) and \((s + 2)\)

Answer: \(s^3 + 11s^2 + 36s + 36\)

**deconv(z,y) Function**

- MATLAB can divide a polynomial \(z(s)\) by another polynomial \(y(s)\) by the command `deconv(z,y)`
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- The syntax \([x,r] = \text{deconv}(z,y)\) is preferred.

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- The syntax \([x,r] = \text{deconv}(z,y)\) is preferred.
- Otherwise MATLAB may show the quotient only and skip the remainder.
**deconv(z,y) Function**

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- The syntax `[x,r] = deconv(z,y)` is preferred.
- Otherwise MATLAB may show the quotient only and skip the remainder.
- The division is shown in two row matrix. One for quotient and one for remainder.
Evaluate the division of 
\((s^2 - 1)\) by \((s + 1)\)

Answer: \((s - 1) + 0\)
Curve fitting: polyfit function

From Data to Equation.

- MATLAB can form a polynomial $c$ that fits the given data for vectors $x$ and $y$ with the command `polyfit(x,y,k)`
Curve fitting: polyfit function

From Data to Equation.

- MATLAB can form a polynomial \( c \) that fits the given data for vectors \( x \) and \( y \) with the command \texttt{polyfit(x,y,k)}
- The syntax \( c = \texttt{polyfit(x,y,k)} \) is preferred.

- \( x, y \) specifies the vectors of the points available for curve fitting.
Curve fitting: \textit{polyfit} function

From Data to Equation.

\begin{itemize}
  \item \textbf{MATLAB} can form a polynomial \( c \) that fits the given data for vectors \( x \) and \( y \) with the command \texttt{polyfit(x,y,k)}
  \item The syntax \( c = \text{polyfit}(x,y,k) \) is preferred.
  \item \( x,y \) specifies the vectors of the points available for curve fitting.
  \item \( k \) specifies the order of the desired polynomial.
\end{itemize}
Find a second order polynomial to predict the discharge (Q) of a pump after 1.5 seconds from the given data.

<table>
<thead>
<tr>
<th>t(s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(L/s)</td>
<td>1</td>
<td>6</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

Answer:

1. t=[0 1 2 4];
2. Q=[1 6 20 100];
3. c=polyfit(t,Q,2)
4. MATLAB returns, $c = 7.3409 - 4.8409t + 1.6818$
5. $\therefore Q = 7.3409t^2 - 4.8409t + 1.6818$ is the equation of discharge.
6. Use `polyval(c,1.5)` to get the answer 10.9375 L/s.
MATLAB can differentiate a polynomial $y(s)$ analytically by the command \texttt{polyder(y)}

- The differentiation is shown in a row matrix.
**Advanced Polynomial Operations**

- **MATLAB** can differentiate a polynomial $y(s)$ analytically by the command `polyder(y)`
- The differentiation is shown in a row matrix.
- **MATLAB** can integrate a polynomial $y(s)$ and integration constant $k$ analytically by the command `polyint(y,k)`
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Advanced Polynomial Operations

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Practice this commands using the polynomial $s^4 + 4s^3 + 8s^2 + 16$

---

**SIMULINK**

- **Simulink** is a software package and is a graphical extension of **MATLAB**.
- It is very useful for modeling, simulating and analyzing dynamic systems due to its GUI.
- **Start Simulink**
- **Type Simulink ←**
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SIMULINK Library Browser

Note that there are many blocks to conveniently model dynamic systems.

Frequently used blocks are collected under Commonly Used Blocks.

In this course we’ll be using blocks from the following

1. Continuous
2. Discontinuous
3. Math Operations
4. Sinks
5. Sources
6. Signal Routing
7. Ports and subsystems
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**Basic Math Operations**

**Demonstration**

Scope Vs. Display

1. Add two numbers and check the output in both Scope and Display
2. Subtract two numbers and check the output in both Scope and Display
3. Multiply two numbers and check the output in Scope and Display
4. Divide two numbers and check the output in Scope and Display

**Demonstration**

View the response of $50 + 60 \sin(0.25t + \phi)$

**Demonstration**

View the response of $50 + 60 \sin(0.25t + \phi)$ using ‘add’ and ‘gain’ operator.
Basic Math Operations

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Demonstration
View the response of $50 + 60\sin(0.25t + \phi)$ using 'add' and 'gain' operator.

Show the three input in a single graph.

- $100 + \sin(0.25t + 2)$
- $50 + \sin(0.5t)$
- $75 + \sin(0.75t + 1)$
Simulink can solve both linear and nonlinear system of equations.

System of equations
\[
\begin{align*}
2x + 3y &= 13 \\
5x - y &= 7
\end{align*}
\]

Write the equations in the following form:

\[
2x + 3y = 13
\]

\[
\Rightarrow x = \frac{13 - 3y}{2}
\]

\[
5x - y = 7
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\[
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\[ \Rightarrow x = \frac{13 - 3y}{2} \]

\[ 5x - y = 7 \]

\[ \Rightarrow y = 5x - 7 \]
Solve the following system.

System 1
\[
\begin{align*}
x + 3y + z &= 5 \\
2x - y + 2z &= 17 \\
3x + 4y + 5z &= 26
\end{align*}
\]

System 2
\[
\begin{align*}
a + b + c &= 6 \\
2a + 3b + 4c &= 15 \\
a + 2b - c &= 10
\end{align*}
\]
Solution of ODE

Demonstration

If \( \frac{dy}{dt} = 0.5t^2 \) find \( y \).
\( y(0) = 0 \)

Demonstration

\( \frac{dx}{dt} - 3t^2 = 2t + \sin(0.25t) \); \( x(0) = 0 \)

General System Modelling

A dynamic system can be represented in general by the differential equation:
A dynamic system can be represented in general by the differential equation:

\[
a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = F(t)
\]

- \( F(t) \) → Forcing Function
- \( x(t) \) → Output or the response of the system
- a's → Constants, System Parameters

Order of a system is designated by order of the differential equation.
Zeroth Order System

\[ x = kF(t) \]

- \( k = \frac{1}{a_0} \leftrightarrow \)
  - Static sensitivity or gain. It represents scaling between the input and the output.
Zeroth Order System

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Output follows the input without distortion or time lag.
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- \( k = \frac{1}{a_0} \leftrightarrow \) Static sensitivity or gain. It represents scaling between the input and the output.
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- Output follows the input without distortion or time lag.
- System requires no additional dynamic considerations.
Zeroth Order System

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- \( k = \frac{1}{a_0} \); Static sensitivity or gain. It represents scaling between the input and the output.
- No equilibrium seeking force is present.
- Output follows the input without distortion or time lag.
- System requires no additional dynamic considerations.
- Represents ideal dynamic performance.
- Example: Potentiometer, ideal spring etc.

First Order System

\[ a \frac{dq(t)}{dt} + bq(t) = u(t) \]
First Order System

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- \( a, b \) are constants or physical system parameters.
- \( u(t) \) is the input or forcing function.
First Order System

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- \(a, b\) are constants or physical system parameters.
- \(u(t)\) is the input or forcing function.
- \(q(t)\) is the output or response of the system.

May be written as

\[ \tau \frac{dq(t)}{dt} + q(t) = \frac{u(t)}{b} \]
Consider, a thermocouple initially at temperature $T$, exposed to ambient temperature $T_\alpha$.

\[ \dot{Q}_{in} = hA(T_\alpha - T) \]
Consider, a thermocouple initially at temperature $T$, exposed to ambient temperature $T_\alpha$.

1. **Convection Heat Transfer**: 
   $$\dot{Q}_{\text{in}} = hA(T_\alpha - T)$$

2. **Heat Transferred to Solid**: 
   $$\dot{Q}_{\text{out}} = mC_v \frac{dT}{dt}$$

3. **First Law of TD**: 
   $$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}}$$
Consider, a thermocouple initially at temperature $T$, exposed to ambient temperature $T_\alpha$.

1. Convection Heat Transfer: $\dot{Q}_{in} = hA(T_\alpha - T)$
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3. First Law of TD $\Rightarrow \dot{Q}_{in} = \dot{Q}_{out}$

4. $\Rightarrow hA(T_\alpha - T) = mC_v \frac{dT}{dt}$

5. $\tau \frac{dT}{dt} + T = T_\alpha$

6. Where $\tau = \frac{mC_v}{hA}$
Time Constant, $\tau$ - time required to complete 63.2% of the process.

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- Rise Time, $T_r$ - time required to achieve response from 10% to 90% of final value.
First Order System

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- **Rise Time**, \( T_r \) - time required to achieve response from 10% to 90% of final value.
- For first order system, \( T_r = 2.31\tau - 0.11\tau = 2.2\tau \).

- **Settling Time**, \( T_s \) - the time for the response to reach, and stay within 2% of its final value.
**First Order System**

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- Process is assumed to be completed when $\tau \geq 5\tau$. 
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- Faster response is associated with shorter $\tau$. 

---

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---

**ME 3204: Control Engineering Sessional**  
Spring 2016  
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**Transfer Function**

- **Transfer function** of a system, $G(s)$, is defined as the ratio of the Laplace Transform (LT) of the output variable, $X(s)$, to the LT of the input variable, $F(s)$, with all the initial conditions are assumed to be zero.

\[ G(s) = \frac{X(s)}{F(s)} \]

- $s = \sigma + j\omega$
Transfer Function

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\[
G(s) = \frac{X(s)}{F(s)}
\]

- \( s = \sigma + j\omega \)
- Amplitude Ratio, \( G_a = \text{mod}(G(j\omega)) \)

Transfer Function of a 1st Order System

- For first order system, \( G(s) = \frac{k}{s + 1} \).
Transfer Function of a 1st Order System

- For first order system, \( G(s) = \frac{k}{\tau s + 1} \).
- Try the derivation

\[ \tau \frac{dx(t)}{dt} + x(t) = kF(t) \]
Transfer Function of a 1st Order System

- For first order system, \( G(s) = \frac{k}{sT + 1} \).
- **Try the derivation**

\[ \tau \frac{dx(t)}{dt} + x(t) = kF(t) \]

Apply Laplace Transform to both sides:

\[ \frac{d^n x(t)}{dt^n} \Rightarrow s^n X(s) \& F(t) \Rightarrow F(s) \]
For first order system, \( G(s) = \frac{k}{s+1} \).

Try the derivation

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\tau \frac{dx(t)}{dt} + x(t) = kF(t)
\]

Apply Laplace Transform to both sides

\[
\frac{d^n x(t)}{dt^n} \Rightarrow s^n X(s) \quad \& \quad F(t) \Rightarrow F(s)
\]

\[
\Rightarrow \tau s X(s) + X(s) = kF(s)
\]

\[
\Rightarrow (\tau s + 1) X(s) = kF(s)
\]
Transfer Function of a 1st Order System

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\frac{d^n x(t)}{dt^n} \Rightarrow s^n X(s) \quad \text{&} \quad F(t) \Rightarrow F(s)
\]

\[
\Rightarrow \tau s X(s) + X(s) = kF(s)
\]

\[
\Rightarrow (\tau s + 1)X(s) = kF(s)
\]

\[
\Rightarrow \frac{X(s)}{F(s)} = G(s) = \frac{k}{\tau s + 1}
\]

Second Order System

Write Newton’s 2nd Law for the system.

\[\sum F = ma\]
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\[ \sum F = ma \]

\[ \Rightarrow F - cv - kx = ma \]

\[ \Rightarrow ma + cv + kx = F \]
Write Newton's 2nd Law for the system.

\[
\sum F = ma
\]

\[\Rightarrow F - cv - kx = ma\]

\[\Rightarrow ma + cv + kx = F\]

\[\Rightarrow m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F\]

Substitute the following:

\[\omega_n = \sqrt{\frac{k}{m}} = \text{Undamped Natural Frequency (rad/s)}\]

\[C_c = 2\sqrt{mk} = \text{Critical Damping Coefficient}\]

\[\zeta = \frac{c}{2C_c} = \text{Damping Ratio}\]
TF of Second Order System

\[ m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = F(t) \]

\[ \Rightarrow \frac{1}{\omega_n^2} \frac{d^2 x}{dt^2} + 2 \zeta \frac{dx}{dt} + x = \frac{F(t)}{k} \]
\[ m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = F(t) \]

\[ \Rightarrow \frac{1}{\omega_n^2} \frac{d^2 x}{dt^2} + 2 \zeta \frac{dx}{\omega_n dt} + x = \frac{F(t)}{k} \]

Taking Laplace Transform of both sides.

\[ \frac{d^n x(t)}{dt^n} \Rightarrow s^n X(s) \quad \text{&} \quad F(t) \Rightarrow F(s) \]

\[ \Rightarrow \frac{1}{\omega_n^2} s^2 X(s) + 2 \zeta sX(s) + X(s) = \frac{F(s)}{k} \]
TF of Second Order System

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Taking Laplace Transform of both sides.

\[ \frac{d^n x(t)}{dt^n} \Rightarrow s^n X(s) & F(t) \Rightarrow F(s) \]

\[ \Rightarrow \frac{1}{\omega_n^2} s^2 X(s) + 2 \frac{\zeta}{\omega_n} s X(s) + X(s) = \frac{F(s)}{k} \]

TF of Second Order System (contd.)

\[ G(s) = \frac{X(s)}{F(s)} = \frac{1}{k} \frac{1}{s^2 + 2 \frac{\zeta}{\omega_n} s + 1} \]
\[ G(s) = \frac{X(s)}{F(s)} = \frac{1}{k} \frac{1}{\frac{1}{\omega_n^2} s^2 + 2 \zeta \omega_n s + 1} \]

\[ \Rightarrow G(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} \]
Response of a 2nd Order System: Step Input

- Steady state position is obtained after a long period of time.
Steady state position is obtained after a long period of time.

**Underdamped System** ($\zeta < 1$): response overshoots the steady-state value initially, & then eventually decays to the steady state value. The smaller $\zeta$ become, the larger is the overshoot.

- Steady state position is obtained after a long period of time.
- **Underdamped System** ($\zeta < 1$): response overshoots the steady-state value initially, & then eventually decays to the steady state value. The smaller $\zeta$ become, the larger is the overshoot.
- **Critically Damped System** ($\zeta = 1$): an exponential rise occurs to approach the steady state value without any overshoot.
Steady state position is obtained after a long period of time.

**Underdamped System** \((\zeta < 1)\): response overshoots the steady-state value initially, & then eventually decays to the steady state value. The smaller \(\zeta\) become , the larger is the overshoot.

**Critically Damped System** \((\zeta = 1)\): an exponential rise occurs to approach the steady state value without any overshoot.

**Overdamped System** \((\zeta > 1)\): response of the system approaches the steady state value without overshoot, but at a slower rate.
1. Consider TF of a system, \( G(s) = \frac{36}{s^2 + 4s + 36} \) with gain \( k = 1 \). Check the response in Simulink.

\[
\omega_n \approx 6 \quad \text{and} \quad \zeta \approx 0.35
\]

Check the underdamped system response.
1. Consider TF of a system, \( G(s) = \frac{36}{s^2 + 4.2s + 36} \) with gain \( k = 1 \). Check the response in Simulink.

\( \omega_n \Rightarrow 6 \) & \( \zeta \Rightarrow 0.35 \)
Check the underdamped system response.

2. Consider TF of a system, \( G(s) = \frac{36}{s^2 + 42s + 36} \) with \( k = 1 \). Check the response in Simulink.

\( \omega_n \Rightarrow 6 \) & \( \zeta \Rightarrow 3.5 \)
Check the overdamped system response.
Practice

1. Consider TF of a system, $G(s) = \frac{36}{s^2 + 4.2s + 36}$ with gain $k = 1$. Check the response in Simulink.

$\omega_n \Rightarrow 6 \& \zeta \Rightarrow 0.35$
Check the underdamped system response.

2. Consider TF of a system, $G(s) = \frac{36}{s^2 + 42s + 36}$ with $k = 1$. Check the response in Simulink.

$\omega_n \Rightarrow 6 \& \zeta \Rightarrow 3.5$
Check the overdamped system response.

3. Consider TF of a system, $G(s) = \frac{36}{s^2 + 12s + 36}$ with $k = 1$. Check the response in Simulink.

$\omega_n \Rightarrow 6 \& \zeta \Rightarrow 1$
Check the critically damped system response.
**Transfer Function in MATLAB**

- MATLAB defines transfer function using the function `tf(n,d)`

- A variable is used to store the transfer function.
Transfer Function in MATLAB

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- \texttt{:.} Syntax is \texttt{variablename=\texttt{tf(n,d)}}

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- \texttt{n} and \texttt{d} are polynomials representing the numerator and denominator of the transfer function.
Transfer Function in MATLAB

- MATLAB defines transfer function using the function \texttt{tf(n,d)}
- A variable is used to store the transfer function.
- ∴ Syntax is \texttt{variable name = tf(n,d)}
- \( n \) and \( d \) are polynomials representing the numerator and denominator of the transfer function.

Define a transfer function, \( G(s) = \frac{5s^2 + 15s + 10}{s^4 + 7s^3 + 20s^2 + 24s} \)

Answer:

1. \( n = [5 \ 15 \ 10] \)
2. \( d = [1 \ 7 \ 20 \ 24 \ 0] \)
3. \( \text{sys} = \text{tf(n,d)} \)
To perform the exact opposite, i.e., separating the numerator and denominator of a given transfer function use the following command.

\[[n,d] = \text{tfdata}(\text{sys}, \text{'v'})\]
To perform the exact opposite, i.e., separating the numerator and denominator of a given transfer function use the following command:

\[
[n,d] = \text{tfdata}(\text{sys}, 'v')
\]

The numerator and denominator will be returned in 'n' and 'd' variables.

'v' is used in syntax to ensure that the polynomials are returned as row vectors and not cell arrays.
Transfer Function in MATLAB

- To perform the exact opposite, i.e., separating the numerator and denominator of a given transfer function use the following command.
- \([n,d] = \text{tfdata}(\text{sys}, 'v')\) ←
- The numerator and denominator will be returned in 'n' and 'd' variables.
- 'v' is used in syntax to ensure that the polynomials are returned as row vectors and not cell arrays.

Practice separating the numerator and Denominator of \(G(s) = \frac{5s^2+15s+10}{s^4+7s^3+20s^2+24s}\)

Practice

Define a transfer function,
\[G(s) = \frac{s(s+1)(s+2)}{s(s+3)(s^2+4s+8)}\]
Define a transfer function, 
\[ G(s) = \frac{s(s+1)(s+2)}{s(s+3)(s^2+4s+8)} \]

Hint: Use poly and conv functions

Transfer Function in MATLAB

- MATLAB can also define transfer function using the following code.
MATLAB can also define transfer function using the following code.

It is helpful when defining TF's with long numerator and denominator.

Define a transfer function, \( G(s) = \frac{s+1}{s-4} \)

---

Answer:

1. \( s = \text{tf('s')} \)
2. \( G = (s+1)/(s-4) \)
A necessary and sufficient condition for a feedback system to be stable is that all the poles of the system transfer function have negative real parts.
Stability

A stable system is a dynamic system with a bounded response to a bounded input. (BIBO)

Zeros and Poles

- Use `tf2zp` to obtain the zeros z, poles p, and gain k of the transfer function, which is defined as a ratio of two polynomials.
Use **tf2zp** to obtain the zeros $z$, poles $p$, and gain $k$ of the transfer function, which is defined as a ratio of two polynomials.

$$ [z,p,k] = \text{tf2zp}(n,d); $$

To obtain the transfer function, as a ratio of two polynomials when zeros $z$, poles $p$ and gain $k$ are known use **zp2tf**.
Use \textbf{tf2zp} to obtain the zeros $z$, poles $p$, and gain $k$ of the transfer function, which is defined as a ratio of two polynomials.

\begin{itemize}
  \item \[ [z, p, k] = \text{tf2zp}(n, d); \]
  \item To obtain the transfer function, as a ratio of two polynomials when zeros $z$, poles $p$ and gain $k$ are known use \textbf{zp2tf}.
  \item \[ [n, d] = \text{zp2tf}(z, p, k); \]
\end{itemize}

The command \textbf{pzmap}(n,d) plots the pole-zero map of a given transfer function.
Zero, Pole, Pole-zero map, Stability

Find the location of zeros and poles and plot the pole-zero map of, \( G(s) = \frac{2s^3 + 8s + 6}{s^4 + 6s^3 + 12s^2 + 24s} \)

Is the system stable?

Answer: In column matrix form, 
\[
z = -3, -1, p = 0, -4.5198, -0.7401 \pm 2.1822i, k = 2. \text{ No.}
\]
Some useful functions for control systems

1. `tf(n,d)` ⇒ Defines transfer functions.
2. `pzmap(sys)` ⇒ Draws the pole-zero map.
Some useful functions for control systems

1. \texttt{tf(n,d)} \Rightarrow \text{Defines transfer functions.}
2. \texttt{pzmap(sys)} \Rightarrow \text{Draws the pole-zero map.}
3. \texttt{bode(sys)} \Rightarrow \text{Draws the bode plot.}
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Some useful functions for control systems

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6. `stepinfo(sys)` ⇒ Rise time, settling time etc.
7. `isstable(sys)` ⇒ Returns 0 if system is unstable.
8. `zpk` ⇒ Creates zero-pole gain model.
9. `impulse(sys)` ⇒ Shows impulse response.
10. `series(sys1,sys2)` ⇒ Equivalent series transfer function.
Some useful functions for control systems

1. \( \text{tf}(n,d) \Rightarrow \) Defines transfer functions.
2. \( \text{pzmap}(\text{sys}) \Rightarrow \) Draws the pole-zero map.
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9. \( \text{impulse}(\text{sys}) \Rightarrow \) Shows impulse response.
10. \( \text{series}(\text{sys1},\text{sys2}) \Rightarrow \) Equivalent series transfer function.
11. \( \text{parallel}(\text{sys1},\text{sys2}) \Rightarrow \) Equivalent parallel transfer function.
12. \( \text{feedback}(\text{sys1},\text{sys2}) \Rightarrow \) Equivalent feedback transfer function.

**TF manipulation: Series**

T \( R(s) \) \( \rightarrow \) \( G_1(s) \) \( \rightarrow \) \( G_2(s) \) \( \rightarrow \) \( G_3(s) \) \( \rightarrow \) \( C(s) \)

\[ X_2(s) = \frac{G_2(s)}{G_1(s)} R(s) \]
\[ X_1(s) = \frac{G_3(s)}{G_2(s) G_1(s)} R(s) \]
\[ C(s) = \frac{G_3(s)}{G_2(s) G_1(s)} R(s) \]

Homework: Take \( G = \frac{1}{s+1} \) and \( H = \frac{1}{s-1} \) to check \( \text{series}(G,H) \).
**TF manipulation: Parallel**

![Diagram of Parallel TF manipulation](image)

- $X_1(s) = R(s)G_1(s)$
- $X_2(s) = R(s)G_2(s)$
- $X_3(s) = R(s)G_3(s)$
- $C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$

Homework: Take $G = \frac{1}{s+1}$ and $H = \frac{1}{s-1}$ to check parallel(G,H).

---

**TF manipulation: Feedback**

![Diagram of Feedback TF manipulation](image)

Homework: Consider,

$$G = \frac{1}{s + 1}$$
$$H = \frac{1}{s - 1}$$

- **Negative feedback system:**
  $$G_{eq} = \frac{G}{1 + GH}$$
  use feedback(G,H)

- **Positive feedback system:**
  $$G_{eq} = \frac{G}{1 - GH}$$
  use feedback(G,H,+1)

*Figure: a) Feedback Control System.
  b) Simplified Model.
  c) Equivalent Transfer Function.

**Closed Loop System H=1**

*same as feedback(G,H,-1)*
Laplace Transform ($\mathcal{L}$) in MATLAB

- Laplace transform helps to convert differential equations which describes the behavior of a dynamic systems into algebraic equations of a complex variable 's'.

- Differentiation and integration are thus replaced by by algebraic operation in complex plane.
Laplace Transform ($\mathcal{L}$) in MATLAB

- Laplace transform helps to convert differential equations which describes the behavior of a dynamic systems into algebraic equations of a complex variable 's'.
- Differentiation and integration are thus replaced by by algebraic operation in complex plane.
- Both transient and steady-state component of the solution are obtained simultaneously.

\[
\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t) \, dt
\]
Laplace Transform ($\mathcal{L}$) in MATLAB

- Laplace transform helps to convert differential equations which describes the behavior of a dynamic systems into algebraic equations of a complex variable 's'.
- Differentiation and integration are thus replaced by algebraic operation in complex plane.
- Both transient and steady-state component of the solution are obtained simultaneously.
- From Definition:

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} [f(t)] \, dt$$

Practice

- Determine the Laplace Transform of $f(t) = e^{-t}(1 - \sin(t))$
Practice

- Determine the Laplace Transform of $f(t) = e^{-t}(1 - \sin(t))$

Use the following command:

```matlab
syms t
```

- Determine the Laplace Transform of $f(t) = e^{-t}(1 - \sin(t))$

Use the following command:

```matlab
syms t
ft = exp(-t) * (1 - sin(t));
```
Practice

- Determine the Laplace Transform of \( f(t) = e^{-t}(1 - \sin(t)) \)

Use the following command:

```matlab
syms t
ft = exp(-t) * (1 - sin(t));
fs = laplace(ft);
```

- Determine the Laplace Transform of \( f(t) = e^{-t}(1 - \sin(t)) \)

Use the following command:

```matlab
syms t
ft = exp(-t) * (1 - sin(t));
fs = laplace(ft);
The result is shown as
fs = 1/(1+s) - 1/(2 + 2s + s^2)
```
Practice

- Determine the Laplace Transform of \( f(t) = e^{-t}(1 - \sin(t)) \)

Use the following command:

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syms t
ft = exp(-t) * (1 - sin(t));
fs = laplace(ft);
The result is shown as
fs = 1/(1+s) - 1/(2 + 2s + s^2)
i.e.,
\[
F(s) = \frac{1}{s+1} - \frac{1}{((s+1)^2+1)}
\]
```

Inverse Laplace Transform \((L^{-1})\) in MATLAB

- Inverse Laplace transform converts the Laplace Transform \(F(s)\) to the time function \(f(t)\).
Inverse Laplace Transform ($L^{-1}$) in MATLAB

- Inverse Laplace transform converts the Laplace Transform $F(s)$ to the time function $f(t)$.
- From Definition:

\[ L^{-1}[F(s)] = f(t) \]

- Determine the Inverse Laplace Transform of $F(s) = \frac{1}{s+4}$
Inverse Laplace Transform ($\mathcal{L}^{-1}$) in MATLAB

- Inverse Laplace transform converts the Laplace Transform $F(s)$ to the time function $f(t)$.
- From Definition:
  $$\mathcal{L}^{-1}[F(s)] = f(t)$$

- Determine the Inverse Laplace Transform of $F(s) = \frac{1}{s+4}$
  ```matlab
  syms s t
  f = 1/(s+4);
  ```

Inverse Laplace Transform ($\mathcal{L}^{-1}$) in MATLAB

- Inverse Laplace transform converts the Laplace Transform $F(s)$ to the time function $f(t)$.
- From Definition:
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  ```matlab
  syms s t
  fs = 1/(s+4);
  ```
Inverse Laplace Transform ($\mathcal{L}^{-1}$) in MATLAB

- Inverse Laplace transform converts the Laplace Transform $F(s)$ to the time function $f(t)$.
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- Determine the Inverse Laplace Transform of $F(s) = \frac{1}{s+4}$

```matlab
syms s t
fs = 1/(s+4);
ft = ilaplace(fs);
```

Inverse Laplace Transform ($\mathcal{L}^{-1}$) in MATLAB

- Inverse Laplace transform converts the Laplace Transform $F(s)$ to the time function $f(t)$.
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- Determine the Inverse Laplace Transform of $F(s) = \frac{1}{s+4}$

```matlab
syms s t
fs = 1/(s+4);
ft = ilaplace(fs);
The result is shown as
ft = exp(-4*t)
```
**Inverse Laplace Transform** ($\mathcal{L}^{-1}$) in **MATLAB**

- Inverse Laplace transform converts the Laplace Transform $F(s)$ to the time function $f(t)$.
- From Definition:
  \[ \mathcal{L}^{-1}[F(s)] = f(t) \]
- Determine the Inverse Laplace Transform of $F(s) = \frac{1}{s+4}$

```matlab
syms s t
fs = 1/(s+4);
ft = ilaplace(fs);
The result is shown as
ft = exp(-4*t)
i.e.,
F(t) = e^{-4t}
```

**Residue Function**

Determine the Partial Fraction expansion of $F(s) = \frac{s^3+9s+1}{s^4+s^3+2s+2}$
Residue Function

Determine the Partial Fraction expansion of \( F(s) = \frac{s^3 + 9s + 1}{s^3 + s^2 + 2s + 2} \)

\[ n = [1 \ 0 \ 9 \ 1]; \leftarrow \]

\[ d = [1 \ 1 \ 2 \ 2]; \leftarrow \]
Residue Function

Determine the Partial Fraction expansion of $F(s) = \frac{s^3 + 9s + 1}{s^3 + s^2 + 2s + 2}$

1. $n = [1 \ 0 \ 9 \ 1]$; ←
2. $d = [1 \ 1 \ 2 \ 2]$; ←
3. $[r, p, k] = \text{residue}(n, d)$ ←

MATLAB outputs the following

$r = 1.0000 \ -1.7678i$
$1.0000 \ + 1.7678i$
$-3.0000$
$p = 0.0000 \ - 1.4142i$
$0.0000 \ + 1.4142i$
$-1$
$k = 1$
Residue Function

MATLAB output:

\[ \begin{align*}
  r &= 1.0000 - 1.7678i \\
  r &= 1.0000 + 1.7678i \\
  r &= -3.0000 \\
  p &= 0.0000 - 1.4142i \\
  p &= 0.0000 + 1.4142i \\
  p &= -1 \\
  k &= 1
\end{align*} \]

\[ \therefore \text{the partial fraction expansion is} \]

\[ F(s) = \frac{s^3 + 9s + 1}{s^3 + s^2 + 2s + 2} = 1 + \frac{1.0000 - 1.7678i}{s - 1.4142i} + \frac{1.0000 + 1.7678i}{s + 1.4142i} - \frac{3}{s + 1} \]
Determine the Partial Fraction expansion of $F(s) = \frac{s+1}{s^4+7s^3+16s^2+12s}$

Answer: 

$$F(s) = \frac{0.6667}{s+3} + \frac{-0.75}{s+2} + \frac{0.5}{(s+2)^2} + \frac{0.0833}{s+0}$$
Find the numerator and denominator of $F(s)$.

1. Define $r, p$ and $k$ first.
2. $[n, d] = \text{residue}(r, p, k) = \leftarrow$
Reverse Residue

\[ F(s) = 1 + \frac{1.0000 - 1.7678i}{s - 1.4142i} + \frac{1.0000 + 1.7678i}{s + 1.4142i} - \frac{3}{s + 1} \]

Find the numerator and denominator of \( F(s) \).

1. define \( r, p \) and \( k \) first.
2. \([n, d] = \text{residue}(r, p, k) = \)
Consider, a simple cruise control system with the assumption that rolling resistance and air drag are proportional to the car's speed \(v\).

**Physical Parameters**
- \(u\) = force generated between the road/tire interface = 500 N
- \(b\) = rolling resistance proportionality constant = 50 Ns/m
- \(m\) = mass of the car = 1000 kg
Car Cruise Control

Consider, a simple cruise control system with the assumption that rolling resistance and air drag are proportional to the car’s speed (v).

- Newton’s 2nd Law: \( \sum F = ma \)

Physical Parameters
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Consider, a simple cruise control system with the assumption that rolling resistance and air drag are proportional to the car’s speed ($v$).

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- $u$ is the force generated between the road/tire interface and can be controlled directly.
- $\Rightarrow ma = u - bv$

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- Newton’s 2nd Law: $\sum F = ma$
- $u$ is the force generated between the road/tire interface and can be controlled directly.
- $ma = u - bv$
- $m \frac{dv}{dt} = u - bv$
- $\Rightarrow \frac{dv}{dt} = \frac{1}{m} \times (u - bv)$

**Physical Parameters**

- $u =$ force generated between the road/tire interface = 500 N
- $b =$ rolling resistance proportionality constant = 50 Ns/m
- $m =$ mass of the car = 1000 kg
Response of a DC motor

- All physical systems can be modeled using differential equations.

[Diagram of a DC motor with labels for voltage, current, and mechanical torque.]
Response of a DC motor

- All physical systems can be modeled using differential equations.
- Response of such systems can be found by simultaneously solving the governing DEQ's.

**Physical Parameters**

- \( J \) = moment of inertia of the rotor (kgm\(^2\))
- \( b \) = motor viscous friction constant (Nms)
- \( K_e \) = electromotive force constant (V/rad/sec)
- \( K_t \) = motor torque constant (Nm/Amp)
- \( R \) = electric resistance (Ohm)
- \( L \) = electric inductance (H)

Response of a DC motor (contd.)

From Newton’s 2nd Law of Motion, \( \sum F = ma \)
Response of a DC motor (contd.)

From Newton’s 2nd Law of Motion, \( \sum F = ma \)
For rotational systems, \( \Rightarrow \sum T = J\alpha \)

\[
\Rightarrow T - b \frac{d\theta}{dt} = J \frac{d^2\theta}{dt^2}
\]
Response of a DC motor (contd.)

From Newton’s 2nd Law of Motion, \[ \sum F = ma \]
For rotational systems, \[ \sum T = J \alpha \]

\[ \Rightarrow T - b \frac{d\theta}{dt} = J \frac{d^2\theta}{dt^2} \]

Motor Torque is proportional to Armature Current.

\[ \therefore T = K_i i \]
From Newton’s 2nd Law of Motion, \( \sum F = ma \)
For rotational systems, \( \Rightarrow \sum T = J\alpha \)

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Motor Torque is proportional to Armature Current.

\[ \therefore T = K_i i \]

\[ \Rightarrow K_i i - \frac{b\theta}{dt} = J\frac{d^2\theta}{dt^2} \]

\[ \therefore \frac{d^2\theta}{dt^2} = \frac{1}{J}(K_i i - \frac{b\theta}{dt}) \quad (1) \]
Response of a DC motor (contd.)

From Kirchoff's Voltage Law, \( \sum V = 0 \)

\[ V = L \frac{di}{dt} + iR + e \]
Response of a DC motor (contd.)

From Kirchhoff's Voltage Law, \( \sum V = 0 \)

\[ \Rightarrow V = L \frac{di}{dt} + iR + e \]

Back emf(e), is proportional to the angular velocity of shaft.

\[ \therefore e = K_e \frac{d\theta}{dt} \]
Response of a DC motor (contd.)

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\[ \Rightarrow V = L \frac{di}{dt} + iR + e \]

Back emf(e), is proportional to the angular velocity of shaft.

\[ \therefore e = K_e \frac{d\theta}{dt} \]

\[ \Rightarrow L \frac{di}{dt} = V - iR - K_e \frac{d\theta}{dt} \]

Solve equation (1) and (2) simultaneously in Simulink for speed/position of motor rotor.
DC Motor Simulink Model

CHANGE SOLVER TO ode15s Simulation Time = 0.2s

Enter in MATLAB
J = 3.2284E-6
b = 3.5077E-6
K = 0.0274
R = 4
L = 2.75E-6
V = 24

Figure: Motor Speed

Figure: Motor Current
Estimating TF (Motor Speed)

Use the following data to estimate TF of motor (I=Voltage, O=Speed).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia of the rotor</td>
<td>0.01 kgm²</td>
</tr>
<tr>
<td>Motor viscous friction constant</td>
<td>0.1 Nms</td>
</tr>
<tr>
<td>Electromotive force constant</td>
<td>0.01 Vradsec</td>
</tr>
<tr>
<td>Motor torque constant</td>
<td>0.01 Nm/Amp</td>
</tr>
<tr>
<td>Electric resistance</td>
<td>1 Ohm</td>
</tr>
<tr>
<td>Electric inductance</td>
<td>0.5 H</td>
</tr>
</tbody>
</table>

1 Declare Subsystem.
2 Set Input and output linearization points. Go to Tools/Control Design/Linear Analysis Tool.
3 Linearize Model. Export model to workspace from LTI viewer.
4 Use the command zpk(modelname).
5 \( \frac{s^2}{(s+9.997)(s+2.003)} \) is the approximate transfer function.

Aircraft Pitch Control
Aircraft Pitch Control

Transfer function, \( G(s) = \frac{\Theta(s)}{\Delta(s)} = ?? \)

Physical Parameters
- \( \alpha \) = Angle of attack
- \( q \) = Pitch Rate
- \( \theta \) = Pitch rate
- \( \delta \) = Elevator deflection

\( \rho \) = Density of air
\( S \) = Platform area of wing
\( \bar{c} \) = Average chord length
\( m \) = Mass of aircraft
\( \mu = \frac{\rho \bar{c} \Delta}{4m} \)
\( U \) = Equilibrium flight speed
\( C_T \) = Coefficient of thrust
\( C_D \) = Coefficient of drag
\( C_L \) = Coefficient of lift
\( C_W \) = Coefficient of weight
\( C_M \) = Coefficient of pitch moment
\( \gamma \) = Flight path angle
\( \Omega = \frac{2U}{\bar{c}} \)
\( \sigma = \frac{1}{1 + \mu C_L} \) = constant
\( i_{yy} \) = Normalized moment of inertia
\( \eta = \mu \sigma C_M \) = constant
Aircraft Pitch Control

Under these assumptions, the following 3 governing equations are used by one of Boeing’s commercial aircraft.

\[
\frac{d\alpha}{dt} = -0.313\alpha + 56.7q + 0.232\delta
\]

\[
\frac{dq}{dt} = -0.0139\alpha - 4.426q + 0.0203\delta
\]

\[
\frac{d\theta}{dt} = 56.7q
\]

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. Therefore, in order to simplify, we will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x- and y-direction and that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit!).

Ans: \( G(s) = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} \)
Aircraft Pitch Control

Under these assumptions, the following 3 governing equations are used by one of Boeing's commercial aircraft.

\[
\frac{d\alpha}{dt} = -0.313\alpha + 56.7q + 0.232\delta
\]

\[
\frac{dq}{dt} = -0.0139\alpha - 4.426q + 0.0203\delta
\]

\[
\frac{d\theta}{dt} = 56.7q
\]

Ans: \( G(s) = \frac{1.151s+0.1774}{s^3+0.739s^2+0.921s} \)

Is the system stable?

Draw step response when elevator deflection is 0.2 rad.

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. Therefore, in order to simplify, we will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x- and y-direction and that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit).
Steady State Error

The difference between the input and output for a prescribed test input as time, t approaches ∞.

Feedback Control

[Diagram of a closed-loop system showing components like input transducer, controller, process or plant, summing junction, and output transducer or sensor.]
In feedback control, the objective is to reduce error signal to zero.

\[ e(t) = y_{sp}(t) - y_m(t) \]
In feedback control, the objective is to reduce error signal to zero.

- $e(t) = y_{sp}(t) - y_{m}(t)$
- $e(t) = \text{Error Signal}$
- $y_{sp}(t) = \text{Set Point}$
In feedback control, the objective is to reduce error signal to zero.

- \( e(t) = y_{sp}(t) - y_{m}(t) \)
- \( e(t) \) = Error Signal
- \( y_{sp}(t) \) = Set Point
- \( y_{m}(t) \) = Measured value of the controlled or process variable (equivalent sensor signal)

Although the error signal equation implies that set point is time varying but in many applications, it is kept constant over a long period of time.
Different types of feedback control

Proportional Control (P)

- For Proportional Control, the objective is to reduce error signal to zero.
- \( p(t) = \bar{p} + K_p e(t) \)
- \( p(t) \) = Controller output
- \( \bar{p} \) = Bias or steady state value
- \( K_p \) = Proportional Controller Gain

Integral Control (I)

- For Integral Control, the rate of change of controller output is proportional to the error signal.
- \( p(t) = \bar{p} + K_i \int e(t) \, dt \) i.e., \( \frac{dp}{dt} = K_i e(t) \)
- \( p(t) \) = Controller output
- \( \bar{p} \) = Bias or steady state value
- \( K_i \) = Integral Controller Gain
Different types of feedback control

Proportional Control ($P$)

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Integral Control ($I$)

- For Integral Control, the rate of change of controller output is proportional to the error signal.
- $p(t) = \bar{p} + K_i \int e(t) \; dt$, i.e., $\frac{dp}{dt} = K_i e(t)$
- $p(t)$ = Controller output
- $\bar{p}$ = Bias or steady state value
- $K_i$ = Integral Controller Gain

Different types of feedback control (contd.)

Derivative Control ($D$)

- For Derivative Control, the controller output is proportional to the rate of change of error signal.
- $p(t) = \bar{p} + K_d \frac{de(t)}{dt}$
- $p(t)$ = Controller output
- $\bar{p}$ = Bias or steady state value
- $K_d$ = Derivative Controller Gain
How PID affects a process/system

<table>
<thead>
<tr>
<th>CL Response</th>
<th>Rise Time</th>
<th>Overshoot</th>
<th>Settling Time</th>
<th>S-S Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small change</td>
<td>Decrease</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Eliminate</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Small change</td>
<td>Decrease</td>
<td>Decrease</td>
<td>No change</td>
</tr>
</tbody>
</table>

- A proportional controller ($K_p$) reduces the rise time and will reduce but never eliminate the steady-state error.
- An integral control ($K_i$) eliminates the steady-state error for a constant or step input, but it may make the transient response slower.
- A derivative control ($K_d$) increases the stability of the system, reducing the overshoot, and improving the transient response.
- Note that these correlations may not be exactly accurate, because $K_p$, $K_i$, and $K_d$ are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for $K_i$, $K_p$ and $K_d$.

PID Controller Design: Step Input, Unit Feedback

```matlab
% Controller Design for Unit feedback
clear all
disp('PID Controller Design for Unit Feedback');
Kp = input('Input Kp = ');
Ki = input('Input Ki = ');
Kd = input('Input Kd = ');

C = tf([Kp Ki 0], [1 0]); % Defines the PID Controller
num = input('Input Plant TF numerator vector = ');
den = input('Input Plant TF denominator vector = ');
P = tf(num, den); % Defines the Process Transfer Function
step(P); % Draws step response of P
hold on % Uses the same figure for next plot
T = feedback(C*P,1); % Unit feedback is fed to the controlled system
t = 0:0.01:1; % Set simulation time
step(T,t); % Draws step response of closed loop feedback system T
```
PID Controller Tuning

Problem

Manually tune a PID controller for a plant having \( G(s) = \frac{1}{s^2 + 10s + 20} \). Your goal is to achieve -

- fast rise time
- minimum overshoot
- no steady-state error

Open Loop Response

- Put \( K_p = 0, K_I = 0, K_D = 0 \) in the code.
- The steady-state error is as much as 95%.
- Settling time is 1.5 sec.
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- Setting time is 1.5 sec.

P-Controller
- Put \( K_P = 300 \) in the code.
- Steady-state error is reduced.
- Rise time is reduced.
- Overshoot increased.
- Setting time slightly reduced.
**PID Controller Tuning (contd.)**

**PD-Controller**
- Put $K_p = 300$ $K_d = 10$ in the code.
- Steady-state error is slightly reduced.
- Rise time is slightly reduced.
- Overshoot reduced.
- Settling time reduced.

---

**PID-Controller**
- Put $K_p = 30$ $K_i = 70$ in the code.
- Proportional gain is reduced because integral controller alone reduces the rise time and increases the overshoot effect.
- If both of them have high values it will create a double effect.
- This controller eliminates the steady-state error.
**PID Controller Tuning (contd.)**

**PD-Controller**
- Put $K_p = 300$ $K_d = 10$ in the code.
- Steady-state error is slightly reduced.
- Rise time is slightly reduced.
- Overshoot reduced.
- Settling time reduced.

**PI-Controller**
- Put $K_p = 30$ $K_i = 70$ in the code.
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- If both of them have high values it will create a double effect.
- This controller eliminates the steady-state error.

**PID-Controller**
- Put $K_p = 350$ $K_i = 300$ $K_d = 5500$ in the code.
- Very fast rise time.
- No overshoot.
- No steady-state error.