CE 416
Prestressed Concrete Sessional
(Lab Manual)

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Preface

The idea of prestressed concrete has been developed around the latter decades of the 19th century, but its use was limited by the quality of the materials at that time. It took until the 1920s and ’30s for its materials development to progress to a level where prestressed concrete could be used with confidence. Currently many bridges and skyscrapers are designed as prestressed structures. This manual intends to provide a general overview about the design procedure of a two way post tensioned slab and a girder. To provide a complete idea, the stress computation, the reinforcement detailing, shear design, the jacking procedure etc. are discussed in details. This Lab manual was prepared with the help of the renowned text book "Design of Prestressed Concrete Structures", 3rd Edition by T.Y. Lin and Ned H. Burns. The design steps for a two way post-tensioned slab was prepared according to the simple hand calculation provided by PCA (Portland Cement Association) as well as the ACI 318-05 code requirements. The design steps for a post-tensioned composite bridge girder were prepared with the help of several sample design calculation demonstrated in different PC structure design books and seminar papers. It has been done in accordance with AASHTO LRFD Bridge Design Specifications.

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INTRODUCTION

Prestressed concrete is a method for overcoming concrete's natural weakness in tension. It can be used to produce beams, floors or bridges with a longer span than is practical with ordinary reinforced concrete. Prestressing tendons (generally of high tensile steel cable or rods) are used to provide a clamping load which produces a compressive stress that balances the tensile stress that the concrete compression member would otherwise experience due to a bending load. Traditional reinforced concrete is based on the use of steel reinforcement bars, rebars, inside poured concrete.

Prestressing can be accomplished in three ways:

- Pre-tensioned concrete,
- Bonded or
- Unbonded post-tensioned concrete.

Pre-tensioned concrete

Pre-tensioned concrete is cast around already tensioned tendons. This method produces a good bond between the tendon and concrete, which both protects the tendon from corrosion and allows for direct transfer of tension. The cured concrete adheres and bonds to the bars and when the tension is released it is transferred to the concrete as compression by static friction. However, it requires stout anchoring points between which the tendon is to be stretched and the tendons are usually in a straight line. Thus, most pretensioned concrete elements are prefabricated in a factory and must be transported to the construction site, which limits their size. Pre-tensioned elements may be balcony elements, lintels, floor slabs, beams or foundation piles.

Bonded post-tensioned concrete

Bonded post-tensioned concrete is the descriptive term for a method of applying compression after pouring concrete and the curing process (*in situ*). The concrete is cast around a plastic, steel or aluminium curved duct, to follow the area where otherwise tension would occur in the concrete element. A set of tendons are fished through the duct and the concrete is poured. Once the concrete has hardened, the tendons are tensioned by hydraulic jacks that react against the concrete member itself. When the tendons have stretched sufficiently, according to the design specifications (see Hooke's law), they are wedged in position and maintain tension after the jacks are removed, transferring pressure to the concrete. The duct is then grouted to protect the tendons from corrosion. This method is commonly used to create monolithic slabs for house construction in locations where expansive soils (such as adobe clay) create problems for the typical perimeter foundation. All stresses from seasonal expansion and contraction of the underlying soil are taken into the entire tensioned slab, which supports the building without significant flexure. Post-tensioning is also used in the construction of various bridges, both after concrete is cured after support by falsework and by the assembly of prefabricated sections, as in the segmental bridge. The advantages of this system over unbonded post-tensioning are:

1. Large reduction in traditional reinforcement requirements as tendons cannot destress in accidents.
2. Tendons can be easily 'weaved' allowing a more efficient design approach.
3. Higher ultimate strength due to bond generated between the strand and concrete.
4. No long term issues with maintaining the integrity of the anchor/dead end.

Unbonded post-tensioned concrete

Unbonded post-tensioned concrete differs from bonded post-tensioning by providing each individual cable permanent freedom of movement relative to the concrete. To achieve this, each individual tendon is coated with a grease (generally lithium based) and covered by a plastic sheathing formed in an extrusion process. The transfer of tension to the concrete is achieved by the steel cable acting against steel anchors embedded in the perimeter of the slab. The main disadvantage over bonded post-tensioning is the fact that a cable can destress itself and burst out of the slab if damaged (such as during repair on the slab). The advantages of this system over bonded post-tensioning are:

1. The ability to individually adjust cables based on poor field conditions (For example: shifting a group of 4 cables around an opening by placing 2 to either side).
2. The procedure of post-stress grouting is eliminated.
3. The ability to de-stress the tendons before attempting repair work.

Applications:

- Prestressed concrete is the predominating material for floors in high-rise buildings and the entire containment vessels of nuclear reactors.
- Unbonded post-tensioning tendons are commonly used in parking garages as barrier cable. Also, due to its ability to be stressed and then de-stressed, it can be used to temporarily repair a damaged building by holding up a damaged wall or floor until permanent repairs can be made.
- The advantages of prestressed concrete include crack control and lower construction costs; thinner slabs - especially important in high rise buildings in which floor thickness savings can translate into additional floors for the same (or lower) cost and fewer joints, since the distance that can be spanned by post-tensioned slabs exceeds that of reinforced constructions with the same thickness. Increasing span lengths increases the usable unencumbered floorspace in buildings; diminishing the number of joints leads to lower

maintenance costs over the design life of a building, since joints are the major focus of weakness in concrete buildings.

- The first prestressed concrete bridge in North America was the Walnut Lane Memorial Bridge in Philadelphia, Pennsylvania. It was completed and opened to traffic in 1951. Prestressing can also be accomplished on circular concrete pipes used for water transmission. High tensile strength steel wire is helically-wrapped around the outside of the pipe under controlled tension and spacing which induces a circumferential compressive stress in the core concrete. This enables the pipe to handle high internal pressures and the effects of external earth and traffic loads.

DESIGN EXAMPLE OF A TWO-WAY POST-TENSIONED SLAB

The following example illustrates the design methods presented in ACI 318-05 and IBC 2003. Unless otherwise noted, all referenced table, figure, and equation numbers are from these books. The example presented here is for Two-Way Post-Tensioned Design**.**

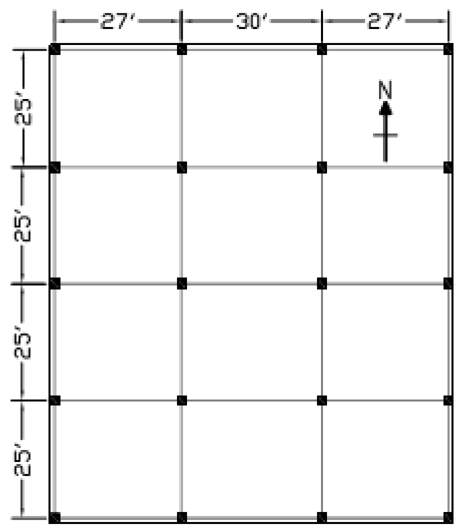

Figure 1 : Typical Plan of a Slab

Loads:

Framing Dead Load = selfweight
Superimposed Dead Load = 25 psf partitions, M/E, misc.
Live Load = 40 psf
residential 2 hour fire-rating
Materials:

Concrete:

Normal weight 150 pcf
f'_c = 5,000 psi
f_{ci} = 3,000 psi

Rebar:

f_y = 60,000 psi
PT: Unbonded tendons 1/2"ϕ, 7-wire strands, A = 0.153 in^2
f_{pu} = 270 ksi
Estimated prestress losses = 15 ksi (ACI 18.6)
f_{se} = 0.7 (270 ksi) - 15 ksi = 174 ksi (ACI 18.5.1)
P_{eff} = A*fse = (0.153)(174 ksi) = 26.6 kips/tendon

Determine Preliminary
Slab Thickness

Start with L/h = 45
Longest span = 30 ft h = (30 ft)(12)/45 = 8.0" preliminary slab thickness

Loading

DL = Selfweight = (8in)(150 pcf) = 100 psf

SIDL = 25 psf
LLo = 40 psf

DESIGN OF EAST-WEST INTERIOR FRAME

Use Equivalent Frame Method, ACI 13.7 (excluding sections 13.7.7.4-5)
Total bay width between centerlines = 25 ft
Ignore column stiffness in equations for simplicity of hand calculations
No pattern loading required, since LL/DL < 3/4 (ACI 13.7.6)

Calculate Section Properties

Two-way slab must be designed as Class U (ACI 18.3.3),
Gross cross-sectional properties allowed (ACI 18.3.4)

$A = bh = (300\ in)(8\ in) = 2{,}400\ in^2$

$S = bh^2/6 = (300\ in)(8\ in)^2/6 = 3{,}200\ in^3$

Set Design Parameters Allowable stresses:

Class U (ACI 18.3.3) At time of jacking (ACI 18.4.1) $f'_{ci} = 3{,}000$ psi

$Compression = 0.60\ f'_{ci} = 0.6(3{,}000\ psi) = 1{,}800\ psi$

$Tension = 3\sqrt{f'_{ci}} = 3\sqrt{3{,}000} = 164\ psi$

At service loads (ACI 18.4.2(a) and 18.3.3)

$f'_c = 5{,}000\ psi$

$Compression = 0.45\ f'_c = 0.45(5{,}000\ psi) = 2{,}250\ psi$

$Tension = 6\sqrt{f'_c} = 6\sqrt{5{,}000} = 424\ psi$

Average precompression limits:

P/A = 125 psi min. (ACI 18.12.4)
300 psi max.

Target load balances:

60%-80% of DL(selfweight) for slabs (good approximation for hand calculation)
For this example: $0.75\ w_{DL} = 0.75(100\ psf) = 75\ psf$
Cover Requirements (2-hour fire rating, assume carbonate aggregate)
IBC 2003

Restrained slabs = 3/4” bottom
Unrestrained slabs = 11/2” bottom
= 3/4” top

Tendon profile:

Parabolic shape; For a layout with spans of similar length, the tendons will be typically be located at the highest allowable point at the interior columns, the lowest possible point at the midspans, and the neutral axis at the anchor locations. This provides the maximum drape for load-balancing.

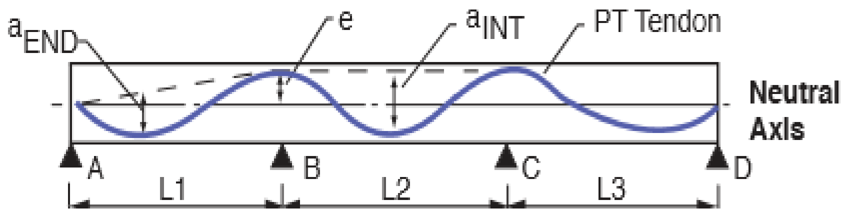

Figure 2: Tendon Profile

Tendon Ordinate	Tendon (CG) Location*
Exterior support - anchor	4.0”
Interior support - top	7.0”
Interior span - bottom	1.0”
End span - bottom	1.75”

(CG) = center of gravity
*Measure from bottom of slab

a_{INT} = 7.0” - 1.0” = 6.0”
a_{END} = (4.0” + 7.0”)/2 - 1.75” = 3.75”
eccentricity, e, is the distance from the center to tendon to the neutral axis; varies along the span

<u>Prestress Force Required to Balance 75% of selfweight DL</u>

Since the spans are of similar length, the end span will typically govern the maximum required post-tensioning force. This is due to the significantly reduced tendon drape, a_{END}.

$w_b = 0.75\ w_{DL}$
= 0.75 (100 psf)(25 ft)
= 1,875 plf
= 1.875 k/ft

Force needed in tendons to counteract the load in the end bay:

$P = w_b L^2 / 8a_{end}$
$= (1.875\text{ k/ft})(27\text{ ft})^2 / [8(3.75\text{ in} / 12)]$
= 547 k

Check Precompression Allowance

Determine number of tendons to achieve 547 k
\# tendons = (547 k) / (26.6 k/tendon)
= 20.56
Use 20 tendons

Actual force for banded tendons
P_{actual} = (20 tendons) (26.6 k) = 532 k

The balanced load for the end span is slightly adjusted w_b = (532/547)(1.875 k/ft) = 1.82 k/ft

Determine actual Precompression stress

$P_{actual} / A = (532\text{ k})(1000) / (2{,}400\text{ in}^2)$

$= 221\text{ psi} > 125\text{ psi min. ok}$

$< 300\text{ psi max. ok}$

Check Interior Span Force

P = (1.875 k/ft)(30 ft)2 / [8(6.0 in / 12)]

= 421 k < 532 k Less force is required in the center bay

For this example, continue the force required for the end spans into the interior span and check the amount of load that will be balanced:

$w_b = (532\text{ k})(8)(6.0\text{ in} /12) / (30\text{ ft})^2$

= 2.36 k/ft wb/wDL = 94%;

This value is less than 100%; acceptable for this design.

East-West interior frame:

Effective prestress force, P_{eff} = 532 kips

Check Slab Stresses

Separately calculate the maximum positive and negative moments in the frame for the dead, live, and balancing loads. A combination of these values will determine the slab stresses at the time of stressing and at service loads.

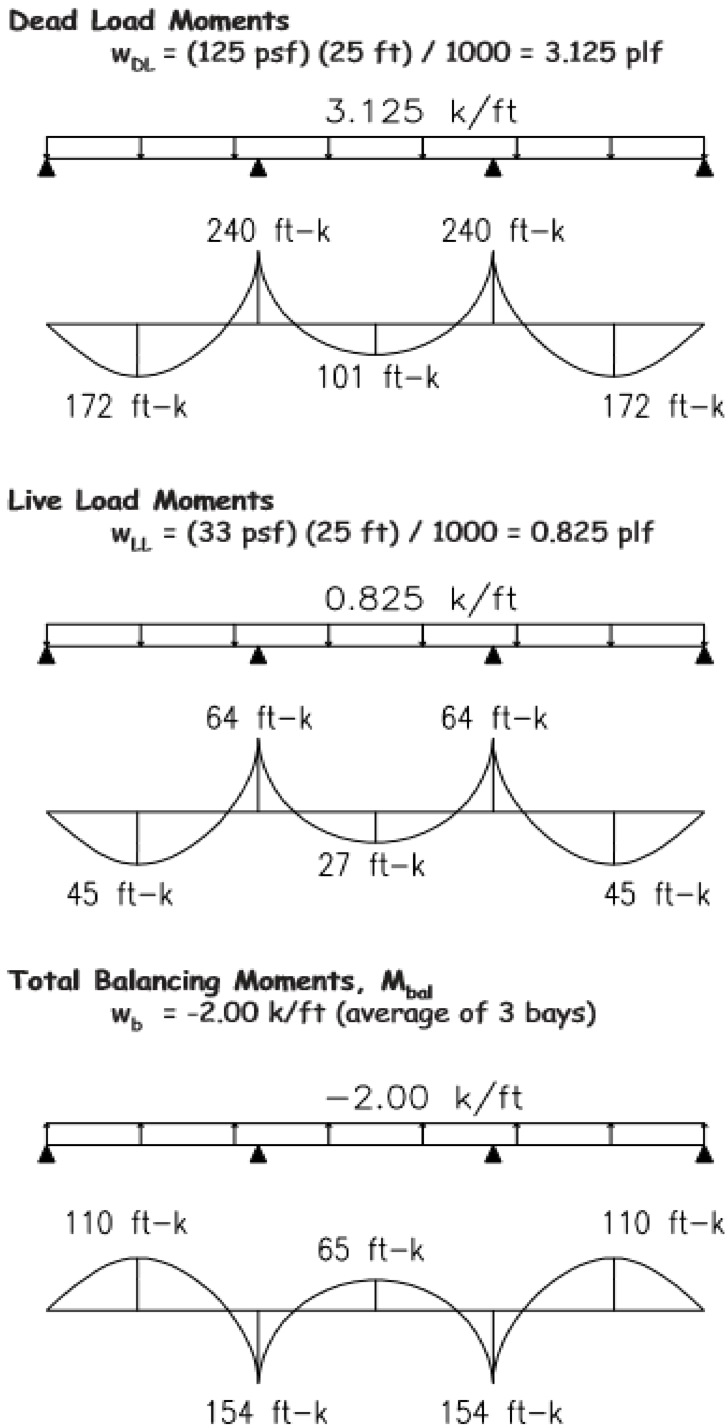

Figure 3: Moment Diagram for DL, LL and Balancing Load

Stage 1: Stresses immediately after jacking (DL + PT) (ACI 18.4.1)

Midspan Stresses

$f_{top} = (-M_{DL} + M_b)/S - P/A$

$f_{bot} = (+M_{DL} - M_b)/S - P/A$

Interior Span

$f_{top} = [(-101\text{ft-k} + 65\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -135 - 221 = -356$ psi compression $< 0.60\ f'_{ci} = 1800$ psi ok

$f_{bot} = [(101\text{ft-k} - 65\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 135 - 221 = -86$ psi compression $< 0.60\ f'_{ci} = 1800$ psi ok

End Span

$f_{top} = [(-172\text{ft-k} + 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -232 - 221 = -453$ psi compression $< 0.60\ f'_{ci} = 1800$ psi ok

$f_{bot} = [(172\text{ft-k} - 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 232 - 221 = 11$ psi tension $< 3\sqrt{f_{ci}} = 164$ psi ok

Support Stresses

$f_{top} = (+M_{DL} - M_b)/S - P/A$

$f_{bot} = (-M_{DL} + M_b)/S - P/A$

$f_{top} = [(240\text{ft-k} - 154\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 323 - 221 = 102$ psi tension $< 3\sqrt{f'_{ci}} = 164$ psi ok

$f_{bot} = [(-240\text{ft-k} + 154\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -323 - 221 = -544$ psi compression $< 0.60\ f'ci = 1800$ psi ok

Stage 2: Stresses at service load (DL + LL + PT) (18.3.3 and 18.4.2)

Midspan Stresses

$f_{top} = (-M_{DL} - M_{LL} + M_b)/S - P/A$

$f_{bot} = (+M_{DL} + M_{LL} - M_b)/S - P/A$

Interior Span

$f_{top} = [(-101\text{ft-k} - 27\text{ft-k} + 65\text{ft-k})(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -236 - 221 = -457$ psi compression $< 0.45\ f'_c = 2250$ psi ok

$f_{bot} = [(101\text{ft-k} + 27\text{ft-k} - 65\text{ft-k})(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 236 - 221 = 15$ psi tension $< 6\sqrt{f'_c} = 424$ psi ok

End Span

$f_{top} = [(-172\text{ft-k} - 45\text{ft-k} + 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= -401 - 221 = -622$ psi compression $< 0.45\ f'_c = 2250$ psi ok

$f_{bot} = [(172\text{ft-k} + 45\text{ft-k} - 110\text{ft-k})(12)(1000)]/(3200\ \text{in}^3) - 221\text{psi}$

$= 401 - 221 = 180$ psi tension $< 6\sqrt{f'_c} = 424$ psi ok

Support Stresses

$f_{top} = (+M_{DL} + M_{LL} - M_b)/S - P/A$
$f_{bot} = (-M_{DL} - M_{LL} + M_b)/S - P/A$

$f_{top} = [(240ft\text{-}k + 64ft\text{-}k - 154ft\text{-}k)(12)(1000)]/(3200\ in^3) - 221psi$
$= 563 - 221 = 342$ psi tension $< 6\sqrt{f'_c} = 424$ psi ok
$f_{bot} = [(-240ft\text{-}k - 64\ ft\text{-}k + 154ft\text{-}k)(12)(1000)]/(3200\ in^3) - 221psi$
$= -563 - 221 = -784$ psi compression $< 0.45\ f'_c = 2250$ psi ok

All stresses are within the permissible code limits.

Ultimate Strength
Determine factored moments
The primary post-tensioning moments, M_1, vary along the length of the span.
$M_1 = P * e$
e = 0 in. at the exterior support
e = 3.0 in at the interior support (neutral axis to the center of tendon)
$M_1 = (532k)(3.0in) / (12) = 133ft\text{-}k$

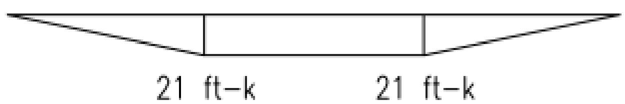

Figure 4: Secondary Moment Diagram

The secondary post-tensioning moments, M_{sec}, vary linearly between supports.
$M_{sec} = M_b - M_1 = 154$ ft-k - 133 ft-k
= 21 ft-k at the interior supports
The typical load combination for ultimate strength design is
$M_u = 1.2\ M_{DL} + 1.6\ M_{LL} + 1.0\ M_{sec}$
At midspan:
$M_u = 1.2$ (172ft-k) + 1.6 (45ft-k) + 1.0 (10.5 ft-k) = 289 ft-k
At support:
$M_u = 1.2$ (-240ft-k) + 1.6 (-64ft-k) + 1.0 (21 ft-k) = -370 ft-k

<u>Determine minimum bonded reinforcement:</u>
to see if acceptable for ultimate strength design.

Positive moment region:

Interior span: $f_t = 15$ psi $< 2\sqrt{f'c} = 2\sqrt{5,000} = 141$ psi
No positive reinforcement required (ACI 18.9.3.1)
Exterior span: $f_t = 180$ psi $> 2\sqrt{f'c} = 2\sqrt{5,000} = 141$ psi

Minimum positive moment reinforcement required (ACI 18.9.3.2)
$y = f_t/(f_t + f_c)h$
$= [(180)/(180+622)](8\ in)$
$= 1.80$ in
$N_c = M_{DL+LL}/S * 0.5 * y * l2$

$= [(172 \text{ ft-k} + 45 \text{ ft-k})(12) / (3{,}200 \text{ in}^3)](0.5)(1.80 \text{ in})(25\text{ft})(12) = 220 \text{ k}$

$A_{s,\,min} = N_c / 0.5fy = (220 \text{ k}) / [0.5(60\text{ksi})] = 7.33 \text{ in}^2$

Distribute the positive moment reinforcement uniformly across the slab-beam width and as close as practicable to the extreme tension fiber.

$A_{s,\,min} = (7.33 \text{ in2})/(25 \text{ ft}) = 0.293\text{in}^2/\text{ft}$

Use #5 @ 12 in. oc Bottom = $0.31 \text{ in}^2/\text{ft}$ (or equivalent)

Minimum length shall be 1/3 clear span and centered in positive moment region (ACI 18.9.4.1)

Negative moment region:

$A_{s,\,min} = 0.00075A_{cf}$ (ACI 18.9.3.3)

Interior supports:

$A_{cf} = \text{max. } (8\text{in})[(30\text{ft} + 27\text{ft})/2, 25\text{ft}]*12$

$As_{,\,min} = 0.00075(2{,}736 \text{ in2}) = 2.05 \text{ in}^2$ = 11 - #4 Top (2.20 in^2)

Exterior supports:

$A_{cf} = \text{max. } (8\text{in})[(27\text{ft}/2), 25\text{ft}]*12$

$A_{s,\,min} = 0.00075(2{,}400 \text{ in}^2) = 1.80 \text{ in}^2$

= 9 - #4 Top (1.80 in2)

Must span a minimum of 1/6 the clear span on each side of support (ACI 18.9.4.2)

At least 4 bars required in each direction (ACI 18.9.3.3)

Place top bars within 1.5h away from the face of the support on each side (ACI 18.9.3.3)

= 1.5 (8 in) = 12 in

Maximum bar spacing is 12" (ACI 18.9.3.3)

Check minimum reinforcement if it is sufficient for ultimate strength

$M_n = (A_sf_y + A_{ps}f_{ps})\,(d-a/2)$

d = effective depth

$A_{ps} = 0.153\text{in}^2*(\text{number of tendons}) = 0.153\text{in}^2*(20 \text{ tendons}) = 3.06 \text{ in}^2$

$f_{ps} = f_{se} + 10{,}000 + (f'_cbd)/(300A_{ps})$ for slabs with L/h > 35 (ACI 18.7.2)

$= 174{,}000\text{psi} + 10{,}000 + [(5{,}000\text{psi})(25\text{ft}*12)d]/[(300)(3.06 \text{ in}^2)]$

$= 184{,}000\text{psi} + 1634d$

$a = (A_sf_y + A_{ps}f_{ps}) / (0.85f'_cb)$

At supports d = 8" - 3/4 " - 1/4" = 7"

$f_{ps} = 184{,}000\text{psi} + 1634(7") = 195{,}438\text{psi}$

$a = [(2.20 \text{ in}^2)(60 \text{ ksi}) + (3.06 \text{ in}^2)(195\text{ksi})]/[(0.85)(5\text{ksi})(25\text{ft}*12)] = 0.57$

$\phi Mn = 0.9\,[(2.20 \text{ in}^2)(60 \text{ ksi}) + (3.06 \text{ in}^2)(195\text{ksi})][7" - (0.57)/2]/12$

$= 0.9\,(728\text{k})(6.72\text{in})/12 = 367 \text{ ft-k} < 370\text{ft-k}$

Reinforcement for ultimate strength requirements governs $As,_{\,reqd} = 2.30\text{in}^2$

12 - #4 Top at interior supports 9 - #4 Top at exterior supports

When reinforcement is provided to meet ultimate strength requirements, the minimum lengths must also conform to the provision of ACI 318-05 Chapter 12. (ACI 18.9.4.3)

At midspan (end span)

d = 8" - 11/2" - 1/4" = 6 1/4"

fps = 184,000psi + 1634(6.25")

= 194,212psi

a= [(7.33 in2)(60 ksi) + (3.06 in^2)(194ksi)]/[(0.85)(5ksi)(25ft*12)] = 0.81

ϕMn = 0.9 [(7.33 in^2)(60 ksi) + (3.06 in^2)(194ksi)][6.25" - (0.81)/2]/12

= 0.9 (1033k)(5.85in)/12 = 453 ft-k > 289 ft-k

Minimum reinforcement ok

#5 @ 12" oc Bottom at end spans

This is a simplified hand calculation for a post-tensioned two-way plate design. A detailed example can be found in the PCA Notes on ACI 318-05 Building Code Requirements for Structural Concrete.

DESIGN EXAMPLE OF A POST-TENSIONED COMPOSITE BRIDGE GIRDER

This example illustrates the design of an interior and exterior beam of a precast prestressed concrete beam bridge using fully prestressed beams with harped bonded strands in accordance with the AASHTO LRFD Bridge Design Specifications, Third Edition, Customary US Units and through the 2005 Interims. The bridge consists of a 120-foot simple span. The bridge profile is shown in Figure 1 and the typical section is shown in Figure 2. The concrete deck is 9 inches thick and the abutments are not skewed.

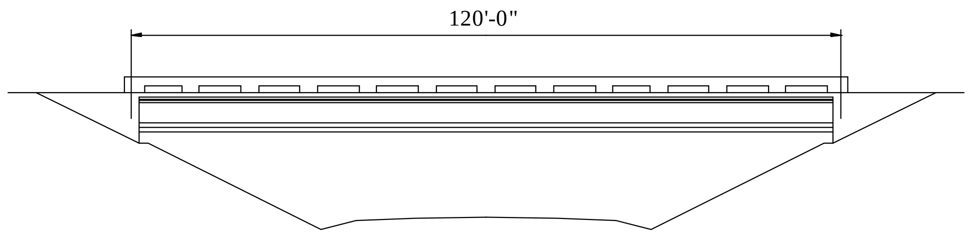

Figure 1 – Profile

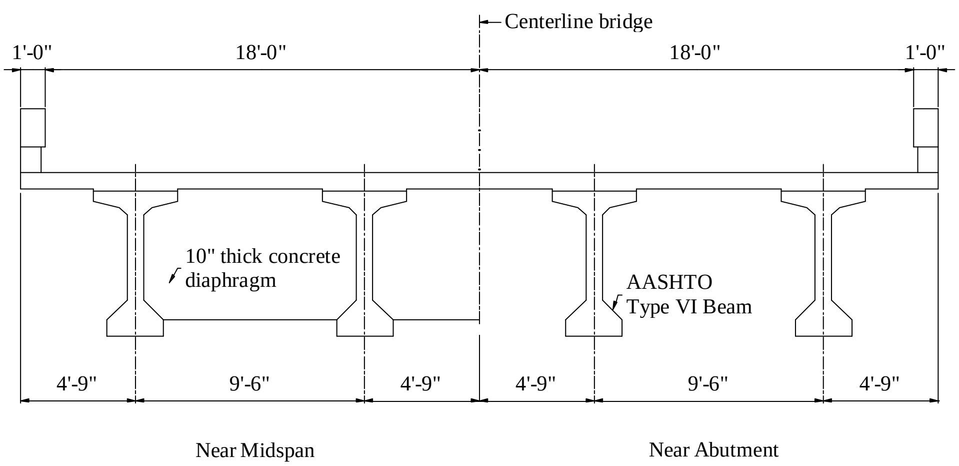

Figure 2 – Typical Section

DESIGN PARAMERS

Corrosion exposure condition for the stress limit for tension in the beam concrete: severe
Provisions for a future wearing surface: 0.025 k/ft^2
Rail dead load per each rail: 0.278 k/foot
Diaphragm dead load per each 10-inch thick diaphragm: 5.590 k
Deck concrete 28-day strength: 5 ksi

$$f_c' = 5\,\text{ksi}$$

$$E_c = 33000w^{1.5}\sqrt{f_c'} = (33000)(0.145)^{1.5}\sqrt{5} = 4074\text{ ksi} \quad (5.4.2.4\text{-}1)$$

Beam concrete 28-day strength: 8 ksi

$$f_c' = 8\,\text{ksi}$$

$$E_c = (33000)(0.145)^{1.5}\sqrt{8} = 5154\text{ ksi}$$

Beam concrete strength at release: 7 ksi

$f'_{ci} = 7$ ksi

$E_{ci} = (33000)(0.145)^{1.5}\sqrt{7} = 4821$ ksi

Non-prestressed reinforcement: Grade 60

$f_y = 60$ ksi

$E_s = 29{,}000$ ksi (5.4.3.2)

Prestressing steel: 0.6-inch diameter, 270-ksi low relaxation strand

$f_{pu} = 270$ ksi

$f_{py} = 0.90f_{pu} = (0.90)(270) = 243$ ksi (Table 5.4.4.1-1)

$E_{ps} = 28{,}500$ ksi (5.4.4.2)

SECTION PROPERTIES

The non-composite and composite section properties are summarized in Table 1. Although the haunch between the top of the girder and the bottom of the deck slab is not included in the composite section properties, it is included in the dead loads. In order to calculate the composite section properties, first calculate the effective flange width.

Effective Flange Width (4.6.2.6.1)

Calculate the effective flange width for the interior beam first. For the interior beam, the effective flange width may be taken as the least of:

a) One-quarter of the effective span length: 120 feet for simple spans

(0.25)(120)(12) = 360 in

b) Twelve time the average thickness of the slab, 9 inches, plus the greater of:

The web thickness: 8 inches

One-half of the top flange of the girder: 42 inches

(0.5)(42) = 21 in

The greater of these two values is 21 inches and:

(12)(9) + 21 = 129 in

c) The average spacing of adjacent beams: 9.5 feet

(9.5)(12) = 114 in

The least of these is 114 inches and therefore, the effective flange width is 114 inches.

For the exterior beam, the effective flange width may be taken as one-half the effective flange width of the adjacent interior beam, 114 inches, plus the least of:

a) One-eight of the effective span length: 120 feet

(0.125)(120)(12) = 180 in

b) Six times the average thickness of the slab, 9 inches, plus the greater of:

One-half of the web thickness: 8 inches

(0.5)(8) = 4 in

One-quarter of the top flange of the girder: 42 inches

(0.25)(42) = 10.5 in

The greater of these two values is 10.5 inches and:

(6)(9) + 10.5 = 64.5 in

c) The width of the overhang: 4.75 feet

(4.75)(12) = 57 in

The least of these is 57 inches and the effective flange width is:

(0.5)(114) + 57 = 114 in

Composite Section Properties

A = area of non-composite beam or deck (in^2)

d = distance between the centers of gravity of the beam or deck and the composite section (in)

I_o = moment of inertia of non-composite beam or deck (in^4)

I_{comp} = moment of inertia of composite section (in^4)

S_b = section modulus of non-composite section, extreme bottom beam fiber (in^3)

S_{bc} = section modulus of composite section, extreme bottom beam fiber (in^3)

$S_{slab\ top}$ = section modulus of composite section, extreme top deck slab fiber (in^3)

S_t = section modulus of non-composite section, extreme top beam fiber (in^3)

S_{tc} = section modulus of composite section, extreme top beam fiber (in^3)

y_b = distance from the center of gravity of the non-composite section to the bottom of the beam (in)

y_{bc} = distance from the center of gravity of the composite section to the bottom of the beam (in)

$y_{slab\ top}$ = distance from the center of gravity of the composite section to the top of the deck slab (in)

y_t = distance from the center of gravity of the non-composite section to the top of the beam (in)

y_{tc} = distance from the center of gravity of the composite section to the top of the beam (in)

w = weight of the non-composite beam (k/ft)

For the composite section, the modular ratio, n, to account for different concrete strengths in the beam and deck slab is:

$$n = \frac{E_{deck}}{E_{beam}} = \frac{4074}{5154} = 0.7906$$

The transformed deck area is:

$$A = (114)(0.7906)(9) = 811.12\ in^2$$

The moment of inertia of the transformed deck area is:

$$I_o = \frac{(114)(0.7906)(9)^3}{12} = 5475\ in^3$$

	A	y_b	Ay_b	d	Ad^2	I_o	$I_o + Ad^2$
Deck	811.12	76.50	62051.00	22.96	427499	5475	432974
Beam	1085.00	36.38	39472.30	17.16	319590	733320	1052910
Total	1896.12		101523.30				1485884

$$y_{bc} = \frac{\sum Ay_b}{\sum A} = \frac{101523.30}{1896.12} = 53.54\ in \qquad S_{bc} = \frac{I_c}{y_{bc}} = \frac{1485884}{53.54} = 27751\ in^3$$

$$y_{tc} = h_{beam} - y_{bc} = 72 - 53.54 = 18.46 \text{ in} \qquad S_{tc} = \frac{I_c}{y_{tc}} = \frac{1485884}{18.46} = 80503 \text{ in}^3$$

$$y_{slab\ top} = y_{tc} + t_{slab} = 18.46 + 9 = 27.46 \text{ in} \qquad S_{slab\ top} = \frac{I_c}{y_{slab\ top}\, n} = \frac{1485884}{(27.46)(0.7906)} = 68443 \text{ in}^3$$

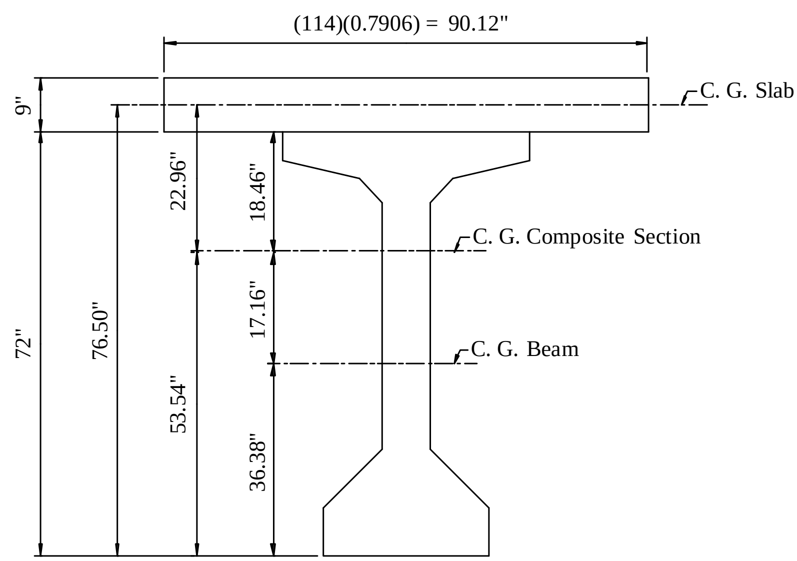

Figure 3 – Composite Section, Interior and Exterior Beams

Table 1 – Section Properties				
Non-composite Section		Composite Section		
Property	Type VI Beams	Property	Interior Beam	Exterior Beam
A (in^2)	1085	I_{comp} (in^4)	1485884	1485884
I (in^4)	733320	y_{bc} (in)	53.54	53.54
y_b (in)	36.38	y_{tc} (in)	18.46	18.46
y_t (in)	35.62	$y_{slab\ top}$ (in)	27.46	27.46
S_b (in^3)	20157	S_{bc} (in^3)	27751	27751
S_t (in^3)	20587	S_{tc} (in^3)	80503	80503
w (k/ft)	1.130	$S_{slab\ top}$ (in^3)	68443	68443

DEAD LOADS

The rail and future wearing surface allowance loads are distributed equally to all beams.

Interior Beam

The dead loads, DC, acting on the non-composite section are:

Beam: 1.130 k/foot

Slab:

(9.5)(0.75)(0.150) = 1.069 k/ft

Haunch: 0.025 k/foot

Diaphragms: 5.590 k at midspan

The dead load, DC, acting on the composite section is:

Rail:

$$\frac{(0.278)(2)}{4} = 0.139\ \text{k/ ft}$$

The dead load, DW, acting on the composite section is:

Future wearing surface allowance (FWS):

$$\frac{(0.025)(36)}{4} = 0.225\ \text{k/ ft}$$

Exterior Beam

The dead loads, DC, acting on the non-composite section are:

Beam: 1.130 k/foot

Slab:

(9.5)(0.75)(0.150) = 1.069 k/ft

Haunch: 0.025 k/foot

Diaphragms: 2.795 k at midspan

The dead load, DC, acting on the composite section is:

Rail: 0.139 k/foot

The dead load, DW, acting on the composite section is:

FWS: 0.225 k/foot

DISTRIBUTION OF LIVE LOAD

Use the approximate formulas found in Article 4.6.2.2 for cross section k, a concrete slab on concrete beams.

K_g = longitudinal stiffness parameter (in^4)

L = span length (ft)

N_b = number of girders

S = girder spacing (ft)

t_s = deck slab thickness (in)

Interior Beam

LONGITUDINAL STIFFNESS PARAMETER (4.6.2.2.1)

e_g = the distance between the centers of gravity of the basic beam and the deck (in)

$$e_g = y_t + \frac{t_s}{2} = 35.62 + \frac{9}{2} = 40.12\ \text{in}$$

The modular ratio for calculating the longitudinal stiffness parameter is:

$$n = \frac{E_B}{E_D} = \frac{5154}{4074} = 1.2651$$

The longitudinal stiffness parameter is:

$$K_g = n\left(I + Ae_g^2\right) = (1.2651)\left[733320 + (1085)(40.12)^2\right] = 3137123\ \text{in}^4$$

DISTRIBUTION OF LIVE LOAD FOR MOMENT (4.6.2.2.2b)
Check the range of applicability.

$3.5 \le S \le 16.0$ $\quad S = 9.5$ feet O.K.

$4.5 \le t_s \le 12.0$ $\quad t_s = 9$ inches O.K.

$20 \le L \le 240$ $\quad L = 120$ feet O.K.

$10{,}000 \le K_g \le 7{,}000{,}000$ $\quad K_g = 3{,}137{,}123 \text{ in}^4$ O.K.

$N_b \ge 4$ $\quad N_b = 4$ O.K.

For one design lane loaded:

$$g = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1} = 0.06 + \left(\frac{9.5}{14}\right)^{0.4}\left(\frac{9.5}{120}\right)^{0.3}\left(\frac{3137123}{(12)(120)(9)^3}\right)^{0.1} = 0.506$$

For the fatigue limit state, remove the multiple presence factor.

$$g = \frac{0.506}{1.2} = 0.422$$

For two or more design lanes loaded:

$$g = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1} = 0.075 + \left(\frac{9.5}{9.5}\right)^{0.6}\left(\frac{9.5}{120}\right)^{0.2}\left(\frac{3137123}{(12)(120)(9)^3}\right)^{0.1} = 0.741$$

DISTRIBUTION OF LIVE LOAD FOR SHEAR (4.6.2.2.3a)
Check the range of applicability.

$3.5 \le S \le 16.0$ $\quad S = 9.5$ feet O.K.

$4.5 \le t_s \le 12.0$ $\quad t_s = 9$ inches O.K.

$20 \le L \le 240$ $\quad L = 120$ feet O.K.

$10{,}000 \le K_g \le 7{,}000{,}000$ $\quad K_g = 3{,}137{,}123 \text{ in}^4$ O.K.

$N_b \ge 4$ $\quad N_b = 4$ O.K.

For one design lane loaded:

$$g = 0.36 + \frac{S}{25.0} = 0.36 + \frac{9.5}{25.0} = 0.740$$

For the fatigue limit state, remove the multiple presence factor:

$$g = \frac{0.740}{1.2} = 0.617$$

For two or more design lanes loaded:

$$g = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0} = 0.2 + \frac{9.5}{12} - \left(\frac{9.5}{35}\right)^{2.0} = 0.918$$

Exterior Beam

DISTRIBUTION OF LIVE LOAD FOR MOMENT (4.6.2.2.2d)
For one design lane loaded, use the lever rule and apply the multiple presence factor, m, because the truck is manually positioned on the bridge cross section. When manually positioning the trucks, the first design lane is placed immediately adjacent to the face of the traffic rail and subsequent lanes are placed immediately adjacent to the previous. The closest truck wheel is placed no closer than two feet from the edge of its design lane. When using the lever rule,

assume hinges at all interior beams and solve for the reaction at the exterior beams. This reaction, when expressed in terms of lanes, is a lane fraction for the exterior beams.

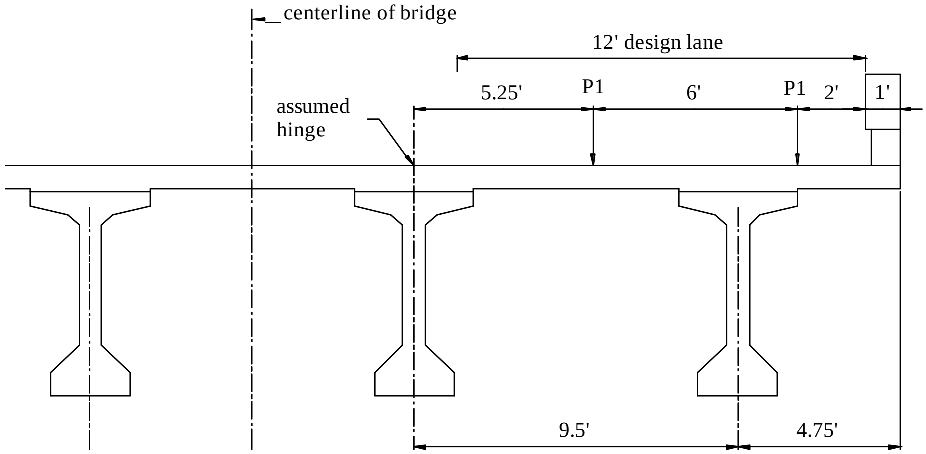

Figure 4 – Lever Rule

Solving for the reaction, R, at the exterior beam:

$$R = \frac{5.25 + 11.25}{9.5} = 1.737 \text{ wheels} = 0.868 \text{ lanes}$$

g = mR = (1.2)(0.868) = 1.042

For the fatigue limit state, g = 0.868

For two or more design lanes loaded, the lane fraction is found by applying a multiplier, e, to the interior beam lane fraction for two or more design lanes loaded. This multiplier is a function of the distance from the exterior web of the exterior beam to the interior edge of the curb or traffic barrier, d_e.

d_e = 4.75 – 1.00 – 0.3333 = 3.4167 ft

Check the range of applicability.

$$-1.0 \le d_e \le 5.5 \qquad d_e = 3.4167 \text{ feet} \quad \text{O.K.}$$

$$e = 0.77 + \frac{d_e}{9.1} = 0.77 + \frac{3.4167}{9.1} = 1.1455$$

$g = e\, g_{interior}$ = (1.1455)(0.741) = 0.848

DISTRIBUTION OF LIVE LOAD FOR SHEAR (4.6.2.2.3b)

For one design lane loaded, use the lever rule. However, this is the same lane fraction for moment, 1.042, and for the fatigue limit state the lane fraction is 0.868. For two or more design lanes loaded, apply a multiplier, e, to the lane fraction for two or more design lanes loaded.

Check the range of applicability.

$$-1.0 \le d_e \le 5.5 \qquad d_e = 3.4167 \text{ feet} \quad \text{O.K.}$$

$$e = 0.6 + \frac{d_e}{10} = 0.6 + \frac{3.4167}{10} = 0.9417$$

$g = e\, g_{interior}$ = (0.9417)(0.918) = 0.864

DISTRIBUTION OF LIVE LOAD FOR MOMENT AND SHEAR

Apply the additional investigation provision, which is for moment (4.6.2.2.2d) and shear (4.6.2.2.3b). Once again, when manually positioning the trucks, place the design lanes in the same manner as for the lever rule and apply the multiple presence factor, m. Use the suggested commentary equation, C4.6.2.2.2d-1.

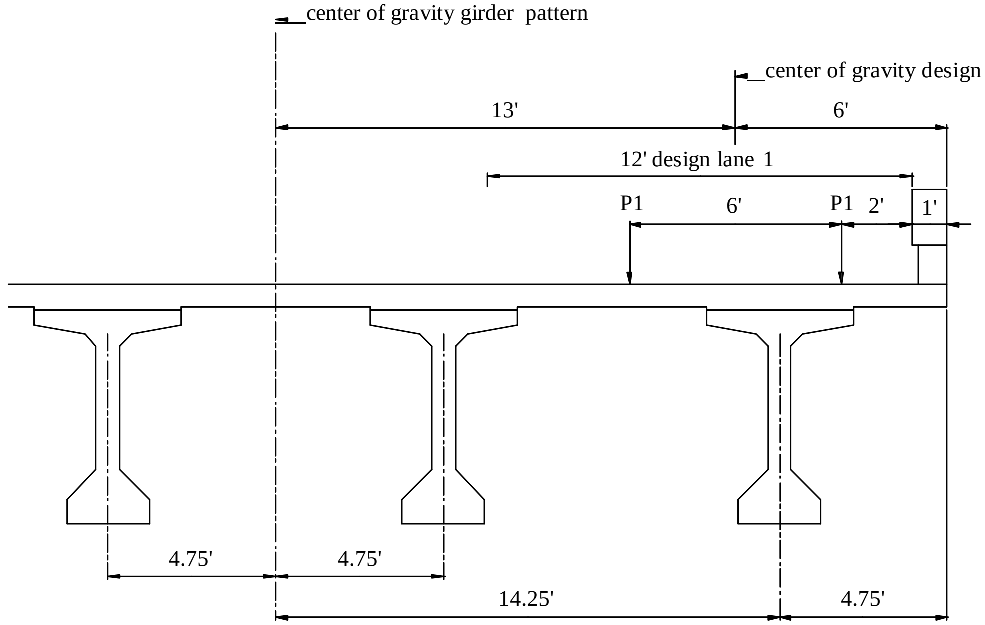

Figure 5 – Rigid Body Rule, One Design Lane Loaded

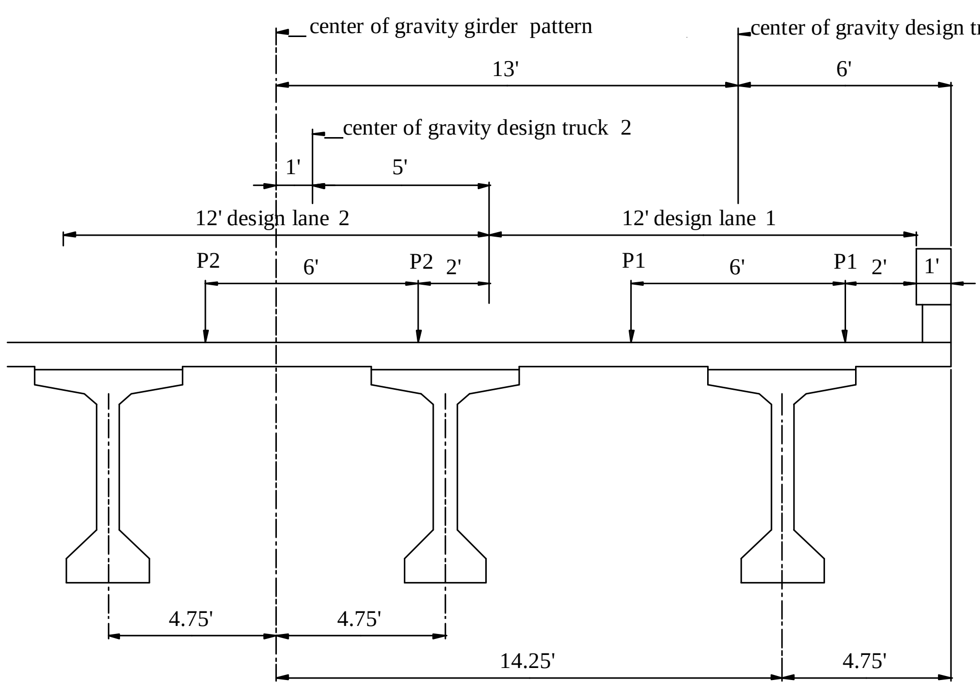

Figure 6 – Rigid Body Rule, Two Design Lanes Loaded

For one design lane loaded:

$$R = \frac{N_L}{N_B} + \frac{x_{ext}\sum e}{\sum x^2} = \frac{1}{4} + \frac{(14.25)(13)}{(2)\left[(4.75)^2 + (14.25)^2\right]} = \frac{1}{4} + \frac{185.25}{451.25} = 0.661$$

g = mR = (1.2)(0.661) = 0.793

For the fatigue limit state, g = 0.661

For two design lanes loaded:

$$R = \frac{2}{4} + \frac{(14.25)(13+1)}{451.25} = 0.942$$

g = mR = (1.0)(0.942) = 0.942

ADJUSTMENT IN LANE FRACTIONS FOR SKEWED BRIDGES

Since the skew angle is zero degrees, there is no adjustment for skew.

SUMMARY OF LANE FRACTIONS

The calculated live load lane fractions are summarized in Table 2. The controlling lane fractions are the largest value for moment and shear for the interior and the exterior beams. The controlling lane fractions are then applied to the appropriate live load envelope for one lane of live load to calculate the design live load moments and shears.

Table 2 – Lane Fractions		
	Moment	Shear
Interior Beam		
One lane loaded	0.506	0.740
Two lanes loaded	0.741	0.918
One lane loaded - fatigue	0.422	0.617
Exterior Beam		
One lane loaded	1.042	1.042
Two lanes loaded	0.848	0.864
One lane loaded - fatigue	0.868	0.868
Additional investigation		
One lane loaded	0.793	0.793
Two lanes loaded	0.942	0.942
One lane loaded - fatigue	0.661	0.661

Table 3 – Controlling Lane Fractions		
	Moment	Shear
Interior Beam		
Service, Strength Limit States	0.741	0.918
Fatigue Limit State	0.422	0.617
Exterior Beam		
Service, Strength Limit States	1.042	1.042
Fatigue Limit State	0.868	0.868

LIMIT STATES

Limit states are groups of events or circumstances that cause structures to be unserviceable. The four limit states are Service, Fatigue, Strength, and Extreme Event. The service limit state (5.5.2) places restrictions on stresses, deformations, and crack width under service conditions. The fatigue and fracture limit state (5.5.3) places restrictions on stress range in reinforcement and prestressing tendons under service conditions. The intent is to limit crack growth under repetitive loads and prevent fracture. The strength limit state (5.5.4) is intended to ensure strength and stability and to resist statistically significant load combinations. The extreme event limit state (5.5.5) is intended to ensure structural survival during extreme events as appropriate to the site and use. Includes earthquake, ice load, collision by vessels and vehicles, and certain hydraulic events (major floods).

The most common limit states for prestressed concrete beam design are:
Strength I - Basic load combination relating to the normal vehicular use of the bridge without wind.

1.25 DC + 1.5 DW + 1.75 (LL + IM)

Strength II - Load combination relating to the use of the bridge by Owner-specified special design vehicles, evaluation permit vehicles, or both without wind.

1.25 DC + 1.5 DW + 1.35 (LL + IM)

Service I - Load combination relating to the normal operational use of the bridge with a 55 MPH wind and all loads taken at their nominal values. Also to control crack width in reinforced concrete structures.

1.0 DC + 1.00 DW + 1.00 (LL + IM)

Service III - Load combination relating only to tension in prestressed concrete superstructures with the object of crack control.

1.0 DC + 1.00 DW + 0.80 (LL + IM)

Fatigue - Fatigue and fracture load combination relating to repetitive gravitational vehicular live load and dynamic response under a single design truck.

0.75 (LL + IM)

BEAM STRESSES

In order to determine the number of required strands, first calculate the maximum tensile stress in the beam for Service III Limit State. The number of required strands is usually controlled by the maximum tensile stress. Provide enough effective prestress force so that the tensile stresses in the beam meet the tensile stress limit. For simple spans beams, the maximum tensile force is at midspan at the extreme bottom beam fibers. The tension service stress at the bottom beam fibers, can be calculated using:

$$f_{bottom} = -\frac{M_{beam} + M_{slab}}{S_b} - \frac{M_{rail} + M_{FWS} + 0.8M_{LL}}{S_{bc}} \qquad \text{(Service III Limit State)}$$

Table 4 – Beam Moments at Midspan (k-ft)					
Beam	Dead Load				Live Load plus Dynamic Load allowance
	Non-composite		Composite		Composite
	Beam	Slab/Diaph	Rail	FWS	HL-93
Interior	2034	2137	250	405	2728
Exterior	2034	2053	250	405	3837

Interior Beam – Stresses due to dead load and live load.

$$f_{bottom} = -\frac{(2034+2137)(12)}{20157} - \frac{[250+405+(0.8)(2728)](12)}{27751} = -3.710\ \text{ksi}(t)$$

Exterior Beam – Stresses due to dead load and live load.

$$f_{bottom} = -\frac{(2034+2053)(12)}{20157} - \frac{[250+405+(0.8)(3837)](12)}{27751} = -4.044\ \text{ksi}(t)$$

PRELIMINARY STRAND ARRANGEMENT

The development of a strand pattern is a cyclic process. Two design parameters need to be initially estimated: the total prestress losses and the eccentricity of the strand pattern at midspan. The total required prestress force can be calculated using:

$$P_e = \frac{f_{ten} - f_{bottom}}{\left(\frac{1}{A} - \frac{e}{S_{bnc}}\right)}$$

f_{ten} = the tensile stress limit (ksi)
Δf_{pT} = the estimated total loss in the prestressing steel stress (ksi)
f_{pj} = the stress in the prestressing steel at jacking (ksi)
f_{pe} = the effective stress in the prestressing steel, after all losses (ksi)
P_e = the effective prestress force, after all losses (k)
e = the eccentricity of the strand pattern (in)

The concrete stress limit for tension, all loads applied, after all losses, and subjected to severe corrosion conditions, Table 5.9.4.2.2-1, is:

$f_{ten} = 0.0948\sqrt{f'_c} = 0.0948\sqrt{8} = 0.268$ ksi
$\Delta f_{pT} = 40$ ksi (estimated)
$f_{pj} = (0.75)(270) = 202.50$ ksi
$f_{pe} = 202.50 – 40 = 162.50$ ksi
$P_e = (f_{pe})(A_{ps}) = (162.50)(0.217) = 35.3$ k (for one strand)
e = -32 inches (estimated)

Interior Beam

$$P_e = \frac{(-0.268)-(-3.710)}{\left(\frac{1}{1085} - \frac{(-32)}{20157}\right)} = 1371.8 \text{ k}$$

The number of strands required is:

$$\frac{1371.8}{35.3} = 38.9$$

Exterior Beam

$$P_e = \frac{(-0.268)-(-4.044)}{\left(\frac{1}{1085} - \frac{(-32)}{20157}\right)} = 1504.9 \text{ k}$$

The number of strands required is:

$$\frac{1504.9}{35.3} = 42.6$$

Try 42 strands. The preliminary strand pattern is shown in Figure 8 and the preliminary strand profile including the eccentricities at the span tenth points is shown in Figure 9. At the midspan of the beam the distance from the bottom of the beam to the center of gravity of the prestressing strands is:

$$\bar{y} = \frac{(13)(2)+(13)(4)+(13)(6)+(3)(8)}{42} = 4.29 \text{ in}$$

The eccentricity of the prestressing strands at the midspan is:

$$e = \bar{y} - y_{bnc} = 4.29 - 36.38 = -32.09 \text{ in}$$

At the ends of the beam the distance from the bottom of the beam to the center of gravity of the prestressing strands, with 12 strands harped at the 0.4 span point, is:

$$\bar{y} = \frac{(10)(2)+(10)(4)+(10)(6)+(3)(64)+(3)(66)+(3)(68)+(3)(70)}{42} = 22.00 \text{ in}$$

The eccentricity of the prestressing strands at the ends of the beam is:

$$e = 22.00 - 36.38 = -14.38 \text{ in}$$

PRESTRESS LOSS – LOW RELAXATION STRAND

In order to check the final beam stresses, calculate the effective prestress force in the prestressing strands. The effective prestress force is the force in the prestressing strands after all prestress losses have occurred. This example assumes that the prestressing strands are jacked to an initial stress of $0.75f_{pu}$ or 202.50 ksi.

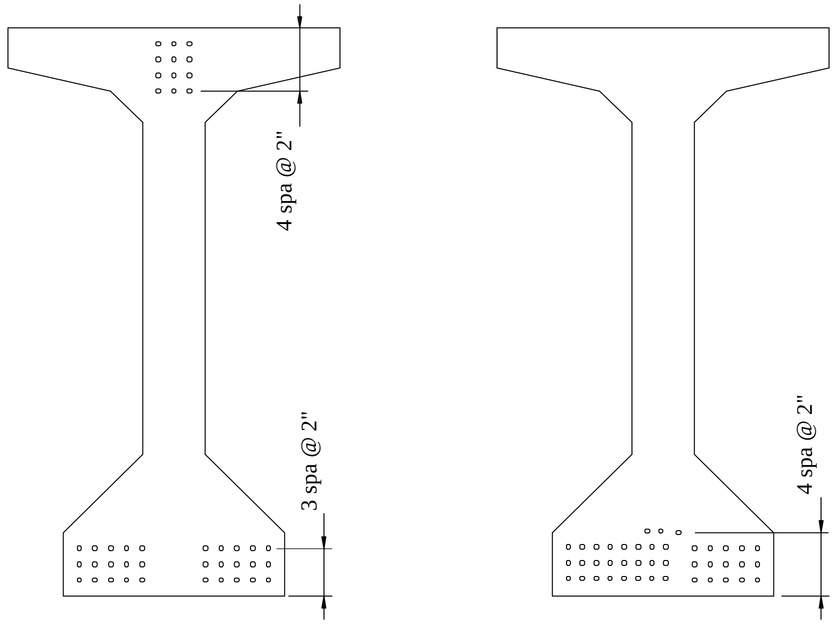

Figure 7 – Strand Pattern

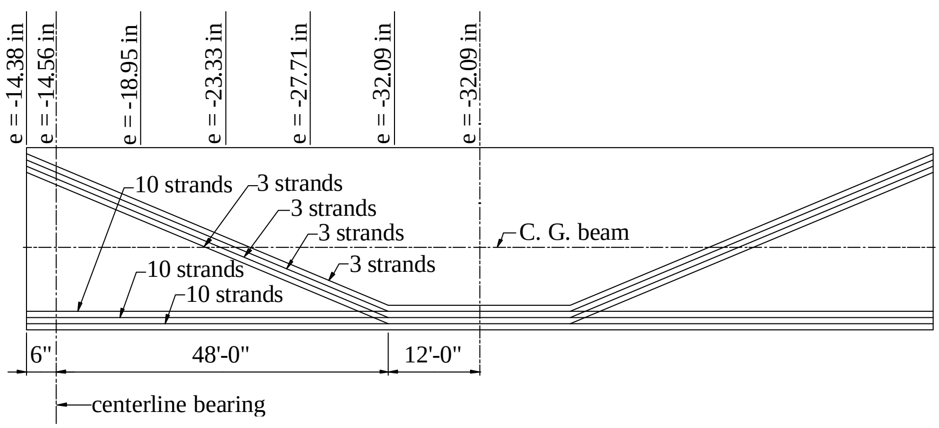

Figure 8 – Strand Profile

GIRDER CREEP COEFFICIENTS

$\Psi_b(t_f,t_i)$ = girder creep coefficient at final time due to loading introduced at transfer per Equation 5.4.2.3.2-1

$\Psi_b(t_d,t_i)$ = girder creep coefficient at time of deck placement due to loading introduced at transfer per Equation 5.4.2.3.2-1

$\Psi_b(t_f,t_d)$ = girder creep coefficient at final time due to loading at deck placement per Equation 5.4.2.3.2-1

H = relative humidity (%)

k_{vs} = factor for the effect of the volume-to-surface ratio of the component

k_f = factor for the effect of concrete strength

k_{hc} = humidity factor for creep

k_{td} = time development factor

t =	maturity of concrete (Day), defined as age of concrete between time of loading for creep calculations, or end of curing for shrinkage calculations, and time being considered for analysis of creep or shrinkage effects
t_i =	age of concrete when load is initially applied, age at transfer (Days)
t_d =	age at deck placement (Days)
t_f =	final age (Days)
V/S =	volume-to-surface ratio
f'_{ci} =	specified compressive strength of concrete at time of prestressing for pretensioned members and at time of initial loading for nonprestressed members. If concrete age at time of initial loading is unknown at design time, f'_{ci} may be taken as 0.80 f'_c (KSI)

Girder creep coefficient for final time due to loading at transfer
H = 70%
V/S = 5.7 in
t_i = 2 Days
t_f = 20000 days

t = 20000 - 2 = 19998 days

$$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - (0.13)(5.7) = 0.7090 \geq 0.0 \quad \text{use } 0.7090$$

$$k_{hc} = 1.56 - 0.008H = 1.56 - (0.008)(70) = 1.0000$$

$$k_f = \frac{5}{1+f'_{ci}} = \frac{5}{1+7} = 0.6250$$

$$k_{td} = \left(\frac{t}{61-4f'_{ci}+t}\right) = \frac{19998}{61-(4)(7)+19998} = 0.9984$$

$$\psi(t_f, t_i) = 1.9 k_{vs} k_{hc} k_f k_{td} t_i^{-0.118} = (1.9)(0.7090)(1.0000)(0.6250)(0.9984)(2)^{-0.118} = 0.7746$$

Girder creep coefficient at time of deck placement due to loading introduced at transfer
t_d = 180 days

t = 180 - 2 = 178 days

$$k_{td} = \left(\frac{t}{61-4f'_{ci}+t}\right) = \frac{178}{61-(4)(7)+178} = 0.8436$$

$$\psi(t_d, t_i) = 1.9 k_{vs} k_{hc} k_f k_{td} t_i^{-0.118} = (1.9)(0.7090)(1.0000)(0.6250)(0.8436)(2)^{-0.118} = 0.6545$$

Girder creep coefficient at final time due to loading at deck placement
t = 20000 - 180 = 19820 days

$$k_{td} = \left(\frac{t}{61-4f'_{ci}+t}\right) = \frac{19820}{61-(4)(7)+19820} = 0.9983$$

$$\psi(t_f, t_d) = 1.9 k_{vs} k_{hc} k_f k_{td} t_i^{-0.118} = (1.9)(0.7090)(1.0000)(0.6250)(0.9983)(180)^{-0.118} = 0.4554$$

DECK CREEP COEFFICIENTS
Deck creep coefficient at final time due to loading at deck placement

V/S = 5.5 in

t = 20000 - 180 = 19820 days

$$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - (0.13)(5.5) = 0.7350 \geq 0.0 \quad \text{use } 0.7350$$

$$k_f = \frac{5}{1+f'_{ci}} = \frac{5}{1+5} = 0.8333$$

$$k_{td} = \left(\frac{t}{61-4f'_{ci}+t}\right) = \frac{19820}{61-(4)(5)+19820} = 0.9979$$

$$\psi(t_f, t_d) = 1.9 k_{vs} k_{hc} k_f k_{td} t_i^{-0.118} = (1.9)(0.7350)(1.0000)(0.8333)(0.9979)(180)^{-0.118} = 0.6292$$

TRANSFORMED SECTION COEFFICIENTS

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time

Transformed section coefficient for time period between transfer and deck placement

e_{pg} = eccentricity of strands with respect to centroid of girder

$$e_{pg} = y_b - \bar{y} = 36.38 - 4.29 = 32.09 \text{ in}$$

$$K_{id} = \frac{1}{1 + \frac{E_p}{E_{ci}}\frac{A_{ps}}{A_g}\left(1 + \frac{A_g e_{pg}^2}{I_g}\right)\left(1 + 0.7\psi_b(t_f, t_i)\right)}$$

$$K_{id} = \frac{1}{1 + \left(\frac{28500}{4821}\right)\left(\frac{9.114}{1085}\right)\left(1 + \frac{(1085)(32.09)^2}{733320}\right)\left(1 + (0.7)(0.7746)\right)} = 0.8380$$

Transformed section coefficient for time period between deck placement and final time

e_{pc} = eccentricity of strands with respect to centroid of composite section

A_c = area of section calculated using the net composite concrete section properties of the girder and the deck and the deck-to-girder modular ratio

I_c = moment of inertia of section calculated using the net composite concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service

$A_c = 1896.12 \text{ in}^2$

$I_c = 1485884 \text{ in}^4$

$$e_{pc} = y_{bc} - \bar{y} = 53.54 - 4.29 = 49.25 \text{ in}$$

$$K_{df} = \frac{1}{1 + \frac{E_p}{E_{ci}}\frac{A_{ps}}{A_c}\left[1 + \frac{A_c e_{pc}^2}{I_c}\right]\left(1 + 0.7\psi_b(t_f, t_i)\right)}$$

$$K_{df} = \frac{1}{1 + \left(\frac{28500}{4821}\right)\left(\frac{9.114}{1896}\right)\left[1 + \frac{(1896)(49.25)^2}{1485884}\right]\left(1 + (0.7)(0.7746)\right)} = 0.8478$$

Exterior Beam

ELASTIC SHORTENING, Δf_{pES} (5.9.5.2.3a)

In order to calculate f_{cgp}, the prestress force at transfer needs to be known. However, the prestress force at transfer depends on the loss that occurs at transfer: elastic shortening. The value of f_{cgp} can be either taken as $0.90f_{pi}$ or calculated using an iterative process.

f_{cgp} = sum of concrete stresses at the center of gravity of the prestressing tendons due to the prestressing force at transfer and the self weight of the member at the sections of maximum moment (ksi)

f_{pt} = the stress in the prestressing tendons at transfer, equal to f_{pj} minus the loss at transfer, elastic shortening (ksi)

P_t = the prestressing force at transfer (k)

For the first iteration, calculate f_{cgp} using a stress in the prestressing steel equal to 0.90 of the stress just before transfer.

$P_t = (0.90)(202.50)(9.114) = 1661.0$ k

$M_{beam} = (2034)(12) = 24408$ k-in

$$f_{cgp} = \frac{P_t}{A} + \frac{(P_t e)y}{I} + \frac{M_{beam}y}{I} = \frac{1661.0}{1085} + \frac{[(1661.0)(-32.09)](-32.09)}{733320} + \frac{(24408)(-32.09)}{733320} = 2.5026 \text{ ksi}$$

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} = \left(\frac{28500}{4821}\right)(2.5026) = 14.79 \text{ ksi}$$

$f_{pt} = 202.50 - 14.79 = 187.71$ ksi

For the second iteration, calculate f_{cgp} using a stress in the prestressing steel of 187.71 ksi.

$P_t = (187.71)(9.114) = 1710.8$ k

$$f_{cgp} = \frac{1710.8}{1085} + \frac{[(1710.8)(-32.09)](-32.09)}{733320} + \frac{(24408)(-32.09)}{733320} = 2.9111 \text{ ksi}$$

$$\Delta f_{pES} = \left(\frac{28500}{4821}\right)(2.9111) = 17.21 \text{ ksi}$$

$f_{pt} = 202.50 - 17.21 = 185.21$ ksi

For the third iteration, calculate f_{cgp} using a stress in the prestressing steel of 185.21 ksi.

$P_t = (185.21)(9.114) = 1688.7$ k

$$f_{cgp} = \frac{1688.7}{1085} + \frac{[(1688.7)(-32.09)](-32.09)}{733320} + \frac{(24408)(-32.09)}{733320} = 2.8597 \text{ ksi}$$

$$\Delta f_{pES} = \left(\frac{28500}{4821}\right)(2.8597) = 16.91\ \text{ksi}$$

f_{pt} = 202.50 - 16.91 = 185.59 ksi
For the fourth iteration, calculate f_{cgp} using a stress in the prestressing steel of 185.59 ksi.
P_t = (185.59)(9.114) = 1691.5 k

$$f_{cgp} = \frac{1691.5}{1085} + \frac{[(1691.5)(-32.09)](-32.09)}{733320} + \frac{(24408)(-32.09)}{733320} = 2.8662\ \text{ksi}$$

$$\Delta f_{pES} = \left(\frac{28500}{4821}\right)(2.8662) = 16.94\ \text{ksi}$$

f_{pt} = 202.50 - 16.94 = 185.56 ksi
For the fifth iteration, calculate f_{cgp} using a stress in the prestressing steel of 185.56 ksi.
P_t = (185.56)(9.114) = 1691.2 k

$$f_{cgp} = \frac{1691.2}{1085} + \frac{[(1691.2)(-32.09)](-32.09)}{733320} + \frac{(24408)(-32.09)}{733320} = 2.8655\ \text{ksi}$$

$$\Delta f_{pES} = \left(\frac{28500}{4821}\right)(2.8655) = 16.94\ \text{ksi}$$

LOSSES: TIME OF TRANSFER TO TIME OF DECK PLACEMENT
SHRINKAGE OF GIRDER CONCRETE, Δf_{pSR} (5.9.5.4.2a)

ε_{bid} = concrete shrinkage strain of girder between the time of transfer and deck placement per Equation 5.4.2.3.3-1

Strain due to shrinkage
k_{hs} = humidity factor for shrinkage (%)

$$k_{hs} = (2.00 - 0.014H) = 2.00 - (0.014)(70) = 1.0200$$

$$\varepsilon_{bid} = k_{vs} k_{hs} k_f k_{td} 0.48x10^{-3} = (0.7090)(1.0200)(0.6250)(0.8436)(0.00048) = 0.00018\ \text{in/in}$$

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id} = (0.00018)(28500)(0.8380) = 4.30\ \text{ksi}$$

CREEP OF GIRDER CONCRETE, Δf_{pCR} (5.9.5.4.2b)

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_d, t_i) K_{id} = \left(\frac{28500}{4821}\right)(2.8655)(0.6545)(0.8380) = 9.29\ \text{ksi}$$

RELAXATION OF PRESTRESSING STRANDS, Δf_{pR1} (5.9.5.4.2c)

f_{pt} = stress in prestressing strands immediately after transfer, taken not less than $0.55f_{py}$ in Equation 4

K_L = 30 for low relaxation strands and 7 for other prestressing steel, unless more accurate manufacturer's data are available

The relaxation loss, Δf_{pR1}, may be assumed equal to 1.2 KSI for low-relaxation strands.

$$\Delta f_{pR1} = \frac{f_{pt}}{K_L}\left(\frac{f_{pt}}{f_{py}} - 0.55\right) = \left(\frac{185.56}{30}\right)\left(\frac{185.56}{243} - 0.55\right) = 1.32\ \text{ksi}$$

LOSSES: TIME OF DECK PLACEMENT TO FINAL TIME

SHRINKAGE OF GIRDER CONCRETE, Δf_{pSD} (5.9.5.4.3a)

ε_{bdf} = shrinkage strain of girder between time of deck placement and final time, calculated by Equation 5.4.2.3.3-1, as $\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid}$

Strain due to shrinkage at final time

$$\varepsilon_{bif} = k_{vs}k_{hs}k_f k_{td} 0.48x10^{-3} = (0.7090)(1.0200)(0.6250)(0.9984)(0.00048) = 0.00022 \text{ in / in}$$

$$\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid} = 0.00022 - 0.00018 = 0.00004 \text{ in/in}$$

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df} = (0.00004)(28500)(0.8478) = 0.97 \text{ ksi}$$

CREEP OF GIRDER CONCRETE, Δf_{pCD} (5.9.5.4.3b)

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight and superimposed loads

Δf_{cd} can be calculated using:

$$\Delta f_{cd} = -\frac{M_{slab}\, y}{I} - \frac{(M_{rail} + M_{WS})\, y_{comp}}{I_{comp}}$$

M_{slab} = (2053)(12) = 24636 k-in

M_{rail} = (250)(12) = 3000 k-in

M_{WS} = (405)(12) = 4860 k-in

$$y = \bar{y} - y_b = 4.29 - 36.38 = -32.09 \text{ in}$$

$$y_{comp} = \bar{y} - y_{bc} = 4.29 - 53.54 = -49.25 \text{ in}$$

$$\Delta f_{cd} = -\frac{(24636)(-32.09)}{733320} - \frac{(3000 + 4860)(-49.25)}{1485884} = 1.3386 \text{ ksi}$$

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp} \left(\psi_b(t_f, t_i) - \psi_b(t_d, t_i)\right) K_{df} + \frac{E_p}{E_c} \Delta f_{cd}\, \psi_b(t_f, t_d) K_{df} \geq 0.0$$

$$\Delta f_{pCD} = \left(\frac{28500}{4821}\right)(2.8655)(0.7746 - 0.6545)(0.8478) + \left(\frac{28500}{5154}\right)(1.3386)(0.4554)(0.8478) = 4.58 \text{ ksi}$$

RELAXATION OF PRESTRESSING STRANDS, Δf_{pR2} (5.9.5.4.3c)

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.32 \text{ ksi}$$

SHRINKAGE OF DECK CONCRETE, Δf_{pSS} (5.9.5.4.3d)

Δf_{pSS} = prestress loss due to shrinkage of deck composite section

Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

ε_{ddf} = shrinkage strain of deck concrete between placement and final time per Equation 5.4.2.3.3-1

A_d = area of deck concrete

E_{cd} = modulus of elasticity of deck concrete

e_d = eccentricity of deck with respect to the transformed gross composite section, taken negative in common construction

$A_d = (114)(9) = 1026\ in^2$
$e_d = y_{bc} - y_{bdeck} = 53.54 - (72 + 4.50) = -22.96\ in$

Strain due to shrinkage

$$\varepsilon_{ddf} = k_{vs}k_{hs}k_f k_{td}\,0.48x10^{-3} = (0.7350)(1.0200)(0.8333)(0.9979)(0.00048) = 0.00030\,in/in$$

$$\Delta f_{cdf} = \frac{\varepsilon_{ddf}A_d E_{cd}}{\left(1+0.7\psi_d(t_f,t_d)\right)}\left(\frac{1}{A_c}+\frac{e_{pc}e_d}{I_c}\right)$$

$$\Delta f_{cdf} = \frac{(0.00030)(1026)(4074)}{1+(0.7)(0.4554)}\left(\frac{1}{1896}+\frac{(49.25)(-22.96)}{1485884}\right) = -0.2221\,ksi$$

$$\Delta f_{pSS} = \frac{E_p}{E_c}\Delta f_{cdf}K_{df}\left(1+0.7\psi_b(t_f,t_d)\right) = \left(\frac{28500}{5154}\right)(-0.2221)(0.8478)\left(1+(0.7)(0.4554)\right) = -1.37\ ksi$$

TOTAL LOSSES
$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$
$\Delta f_{pLT} = (4.30 + 9.29 + 1.32) + (0.97 + 4.58 + 1.32 - 1.37) = 20.41\ ksi$

$\Delta f_{pT} = \Delta f_{pLT} + \Delta f_{pES} = 20.41 + 16.94 = 37.35\ ksi$

Interior Beam
ELASTIC SHORTENING, Δf_{pES} (5.9.5.2.3a)
The loss due to elastic shortening only depends on the moment due to the member weight. Since this is the same for the interior and exterior beams, the loss due to elastic shortening is the same for both beams.
$\Delta f_{pES} = 16.94\ ksi$

LOSSES: TIME OF TRANSFER TO TIME OF DECK PLACEMENT
SHRINKAGE OF GIRDER CONCRETE, Δf_{pSR} (5.9.5.4.2a)
The loss due to shrinkage depends on the relative humidity, the amount of time between the time of transfer and the time of deck placement, and the transformed section coefficient for time period between transfer and deck placement. Since these are the same for the interior and exterior beams, the loss due to shrinkage of girder concrete is the same for both beams.
$\Delta f_{pSR} = 4.30\ ksi$

CREEP OF GIRDER CONCRETE, Δf_{pCR} (5.9.5.4.2b)
The loss due to creep depends on the sum of concrete stresses at the center of gravity of the prestressing tendons due to the prestressing force at transfer and the self weight of the member at the sections of maximum moment, the amount of time between the time of transfer and the time of deck placement, and the transformed section coefficient for time period between transfer and deck placement. Since these are the same for the interior and exterior beams, the loss due to creep of girder concrete is the same for both beams.
$\Delta f_{pCR} = 9.29\ ksi$

RELAXATION OF PRESTRESSING STRANDS, Δf_{pR1} (5.9.5.4.2c)

The loss due to relaxation of prestressing strands depends on the stress in prestressing strands immediately after transfer and the type of prestressing strands used. Since these are the same for the interior and exterior beams, the loss due to relaxation of prestressing strands is the same for both beams.

$\Delta f_{pR1} = 1.32$ ksi

LOSSES: TIME OF DECK PLACEMENT TO FINAL TIME

SHRINKAGE OF GIRDER CONCRETE, Δf_{pSD} (5.9.5.4.3a)

The loss due to shrinkage depends on the relative humidity, the amount of time between the time of deck placement and the final time, and the transformed section coefficient for time period between deck placement and final. Since these are the same for the interior and exterior beams, the loss due to shrinkage of girder concrete is the same for both beams.

$\Delta f_{pSD} = 0.97$ ksi

CREEP OF GIRDER CONCRETE, Δf_{pCD} (5.9.5.4.3b)

$M_{slab} = (2137)(12) = 25644$ k-in

$$\Delta f_{cd} = -\frac{(25644)(-32.09)}{733320} - \frac{(3000+4860)(-49.25)}{1485884} = 1.3827 \text{ ksi}$$

$$\Delta f_{pCD} = \left(\frac{28500}{4821}\right)(2.8655)(0.7746 - 0.6545)(0.8478) + \left(\frac{28500}{5154}\right)(1.3827)(0.4554)(0.8478) = 4.68 \text{ ksi}$$

RELAXATION OF PRESTRESSING STRANDS, Δf_{pR2} (5.9.5.4.3c)

The loss due to relaxation of prestressing strands between the time of deck placement and the final time is equal to the loss between the transfer time and the time of deck placement. Since these are the same for the interior and exterior beams, the loss due to relaxation of prestressing strands is the same for both beams.

$\Delta f_{pR2} = 1.32$ ksi

SHRINKAGE OF DECK CONCRETE, Δf_{pSS} (5.9.5.4.3d)

The loss due to shrinkage of the deck concrete depends on the change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete, section properties, the amount of time between the time of deck placement and the final time, and the transformed section coefficient for time period between deck placement and final. Since these are the same for the interior and exterior beams, the loss due to shrinkage of deck concrete is the same for both beams.

$\Delta f_{pSS} = -1.37$ ksi

TOTAL LOSSES

$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$

$\Delta f_{pLT} = (4.30 + 9.29 + 1.32) + (0.97 + 4.68 + 1.32 - 1.37) = 20.51$ ksi

$\Delta f_{pT} = \Delta f_{pLT} + \Delta f_{pES} = 20.51 + 16.94 = 37.45$ ksi

STRAND ARRANGEMENT

Now that the total loss in the prestressing steel stress has been calculated, check if the preliminary strand pattern and eccentricity are still valid. The calculated effective stress in the prestressing steel is:

f_{pe} = 202.50 – 37.45 = 165.05 ksi for the interior beam

f_{pe} = 202.50 – 37.35 = 165.15 ksi for the exterior beam

Interior Beam

$$P_e = \frac{(-0.268)-(-3.710)}{\left(\frac{1}{1085} - \frac{(-32.09)}{20157}\right)} = 1369.3 \text{ k}$$

The effective prestress force in one strand is:

(165.05)(0.217) = 35.8 k

The number of strands required is:

$$\frac{1369.3}{35.8} = 38.2$$

Exterior Beam

$$P_e = \frac{(-0.268)-(-4.044)}{\left(\frac{1}{1085} - \frac{(-32.09)}{20157}\right)} = 1502.2 \text{ k}$$

The effective prestress force in one strand is:

(165.15)(0.217) = 35.8 k

The number of strands required is:

$$\frac{1502.2}{35.8} = 42.0$$

The preliminary strand arrangement with 42 strands is still good.

SERVICE LIMIT STATE

Now that the release and effective prestress forces are known, the critical beam stresses can be checked for Service I and III Limit States. Check the stresses in the interior and exterior beams for the release and service conditions. The release condition includes loads due the release or transfer of the prestress force and the self-weight of the beam. The service condition includes loads due to the effective prestress force, dead load (beam, concrete deck slab, rails, diaphragms, and future wearing surface allowance, and live load. Calculate the beam stress limits for two conditions: temporary before losses for the release condition and after all losses for the service condition. For the temporary before losses condition (5.9.4.1):

Tension:

$$0.0948\sqrt{f'_{ci}} = 0.0948\sqrt{7} = 0.251 \text{ ksi} > 0.2 \text{ ksi, use } 0.2 \text{ ksi}$$

Compression:

$$0.60f'_{ci} = (0.60)(7) = 4.2 \text{ ksi}$$

At the service limit state after all losses (5.9.4.2):

Tension, severe corrosion conditions:

$0.0948\sqrt{f_c'} = 0.0948\sqrt{8} = 0.268$ ksi

Compression:

Due to effective prestress and permanent loads:

$0.45f_c' = (0.45)(8) = 3.6$ ksi

Due to live load and one-half the sum of effective prestress and permanent loads:

$0.40f_c' = (0.40)(8) = 3.2$ ksi

Due to effective prestress, permanent loads, and transient loads:

$0.6\varphi_w f_c'$

The reduction factor, φ_w, is equal to 1.0 if the flange slenderness ratio is not greater than 15. The slenderness ratio is the ratio of the flange width to depth:

$$\frac{\text{flange width}}{\text{flange depth}} = \frac{114}{9} = 12.7$$

Since this value is not greater than 15, φ_w is equal to 1.0 and the stress limit is:

$0.6\varphi_w f_c' = (0.6)(1.0)(8) = 4.8$ ksi

The stress limits for the concrete deck slab at the service limit state:

Compression:

Due to effective prestress and permanent loads:

$0.45f_c' = (0.45)(5) = 2.25$ ksi

Due to effective prestress, permanent loads, and transient loads:

$0.6\varphi_w f_c' = (0.6)(1.0)(5) = 3.0$ ksi

The total area of prestressing steel, the initial or transfer prestressing force, and the effective prestressing forces are:

$A_{ps} = (0.217)(42) = 9.114\ \text{in}^2$

$P_t = (f_{pt})(A_{ps}) = (185.56)(9.114) = 1691.2$ k

$P_e = (f_{pe})(A_{ps}) = (165.05)(9.114) = 1504.3$ k for the interior beam

$P_e = (f_{pe})(A_{ps}) = (165.15)(9.114) = 1505.2$ k for the exterior beam

STRESSES AT TRANSFER

The release stresses are the same for the interior and exterior beams and can be calculated using:

$$f_{bottom} = \frac{P_t}{A} - \frac{P_t e}{S_{bnc}} - \frac{M_{beam}}{S_{bnc}} \quad \text{(Service I Limit State)}$$

$$f_{top} = \frac{P_t}{A} + \frac{P_t e}{S_{tnc}} + \frac{M_{beam}}{S_{tnc}} \quad \text{(Service I Limit State)}$$

At the harp point due to the initial prestress force and the beam dead load

$M_{beam} = (1986.7)(12) = 23840$ k-in (for 121 foot casting length)

$$f_{bottom} = \frac{1691.2}{1085} - \frac{(1691.2)(-32.09)}{20157} - \frac{23840}{20157} = 3.068\ \text{ksi (c)}$$

$$f_{top} = \frac{1691.2}{1085} + \frac{(1691.2)(-32.09)}{20587} + \frac{23840}{20587} = 0.081\ \text{ksi (c)}$$

At end of the transfer length due to the initial prestress force and the beam dead load
The transfer length is 60 strand diameters or 36 inches. Therefore, the end of the transfer length is 36 inches from the end of the beam or 30 inches from the centerline of bearing, the 0.0208 point. The eccentricity of the prestressing force is:

e = 14.56 + (18.95 – 14.56)(0.208) = 15.47 in

M_{beam} = (200.0)(12) = 2400 k-in (for 121 foot casting length)

$$f_{bottom} = \frac{1691.2}{1085} - \frac{(1691.2)(-15.47)}{20157} - \frac{2400}{20157} = 2.738 \text{ ksi (c)}$$

$$f_{top} = \frac{1691.2}{1085} + \frac{(1691.2)(-15.47)}{20587} + \frac{2400}{20587} = 0.404 \text{ ksi (c)}$$

STRESSES AT SERVICE CONDITION
The service stresses, assuming tension at the bottom beam fibers, can be calculated using:

$$f_{bottom} = \frac{P_e}{A} - \frac{P_e e}{S_{bnc}} - \frac{M_{beam} + M_{slab}}{S_{bnc}} - \frac{M_{rail} + M_{WS} + 0.8M_{LL}}{S_{bc}} \quad \text{(Service III Limit State)}$$

$$f_{top} = \frac{P_e}{A} + \frac{P_e e}{S_{tnc}} + \frac{M_{beam} + M_{slab}}{S_{tnc}} + \frac{M_{rail} + M_{WS} + M_{LL}}{S_{tc}} \quad \text{(Service I Limit State)}$$

Exterior Beam
At end of the transfer length due to the effective prestress force and dead load

M_{beam} = (166.0)(12) = 1992 k-in

M_{slab} = (164.2)(12) = 1970 k-in

M_{rail} = (20.4)(12) = 245 k-in

M_{WS} = (33.0)(12) = 396 k-in

$$f_{bottom} = \frac{1505.2}{1085} - \frac{(1505.2)(-15.47)}{20157} - \frac{1992+1970}{20157} - \frac{245+396}{27751} = 2.323 \text{ ksi (c)}$$

$$f_{top} = \frac{1505.2}{1085} + \frac{(1505.2)(-15.47)}{20587} + \frac{1992+1970}{20587} + \frac{245+396}{80503} = 0.457 \text{ ksi (c)}$$

At midspan due to the effective prestress force and dead load

$$f_{bottom} = \frac{1505.2}{1085} - \frac{(1505.2)(-32.09)}{20157} - \frac{24408+24636}{20157} - \frac{3000+4860}{27751} = 1.067 \text{ ksi (c)}$$

$$f_{top} = \frac{1505.2}{1085} + \frac{(1505.2)(-32.09)}{20587} + \frac{24408+24636}{20587} + \frac{3000+4860}{80503} = 1.521 \text{ ksi (c)}$$

At midspan due to the effective prestress force, dead load, and live load

M_{LL} = (3837)(12) = 46044 k-in

$$f_{bottom} = \frac{1505.2}{1085} - \frac{(1505.2)(-32.09)}{20157} - \frac{24408+24636}{20157} - \frac{3000+4860+(0.8)(46044)}{27751} = -0.260 \text{ ksi(t)}$$

$$f_{top} = \frac{1505.2}{1085} + \frac{(1505.2)(-32.09)}{20587} + \frac{24408+24636}{20587} + \frac{3000+4860+46044}{80503} = 2.093 \text{ ksi (c)}$$

At midspan due to the live load and one-half the effective prestress force and dead load

$$f_{top} = \frac{1}{2}\left(\frac{1505.2}{1085} + \frac{(1505.2)(-32.09)}{20587} + \frac{24408 + 24636}{20587} + \frac{3000 + 4860}{80503}\right) + \frac{46044}{80503} = 1.332 \text{ ksi (c)}$$

Interior Beam

At end of the transfer length due to the effective prestress force and dead load

$M_{slab} = (167.7)(12) = 2012$ k-in

$$f_{bottom} = \frac{1504.3}{1085} - \frac{(1504.3)(-15.47)}{20157} - \frac{1992 + 2012}{20157} - \frac{245 + 396}{27751} = 2.319 \text{ ksi (c)}$$

$$f_{top} = \frac{1504.3}{1085} + \frac{(1504.3)(-15.47)}{20587} + \frac{1992 + 2012}{20587} + \frac{245 + 396}{80503} = 0.459 \text{ ksi (c)}$$

At midspan due to the effective prestress force and dead load

$$f_{bottom} = \frac{1504.3}{1085} - \frac{(1504.3)(-32.09)}{20157} - \frac{24408 + 25644}{20157} - \frac{3000 + 4860}{27751} = 1.015 \text{ ksi (c)}$$

$$f_{top} = \frac{1504.3}{1085} + \frac{(1504.3)(-32.09)}{20587} + \frac{24408 + 25644}{20587} + \frac{3000 + 4860}{80503} = 1.571 \text{ ksi (c)}$$

At midspan due to the effective prestress force, dead load, and live load

$M_{LL} = (2728)(12) = 32736$ k-in

$$f_{bottom} = \frac{1504.3}{1085} - \frac{(1504.3)(-32.09)}{20157} - \frac{24408 + 25644}{20157} - \frac{3000 + 4860 + (0.8)(32736)}{27751} = 0.071 \text{ ksi (c)}$$

$$f_{top} = \frac{1504.3}{1085} + \frac{(1504.3)(-32.09)}{20587} + \frac{24408 + 25644}{20587} + \frac{3000 + 4860 + 32736}{80503} = 1.977 \text{ ksi (c)}$$

At the midspan due to the live load and one-half the effective prestress force and dead load

$$f_{top} = \frac{1}{2}\left(\frac{1504.3}{1085} + \frac{(1504.3)(-32.09)}{20587} + \frac{24408 + 25644}{20587} + \frac{3000 + 4860}{80503}\right) + \frac{32736}{80503} = 1.192 \text{ ksi (c)}$$

Check the stress at the top fibers of the deck slab, interior and exterior beams, at midspan for the concrete compression limit. The composite section properties and loads are used. The service stresses, assuming compression in the extreme top slab fibers, can be calculated using:

$$f_{slab\ top} = \frac{M_{rail} + M_{WS} + M_{LL}}{S_{slab\ top}} \qquad \text{(Service I Limit State)}$$

Interior Beam

At midspan due to the composite dead load

$$f_{slab\ top} = \frac{3000 + 4860}{68443} = 0.115 \text{ ksi (c)}$$

At midspan due to the composite dead load and live load

$$f_{slab\ top} = \frac{3000 + 4860 + 32736}{68443} = 0.593\ \text{ksi (c)}$$

Exterior Beam

At midspan due to the composite dead load

$$f_{slab\ top} = \frac{3000 + 4860}{68443} = 0.115\ \text{ksi (c)}$$

At midspan due to the composite dead load and live load

$$f_{slab\ top} = \frac{3000 + 4860 + 46044}{68443} = 0.788\ \text{ksi (c)}$$

Table 5 – Stress Summary, Interior and Exterior Beams

	Maximum	Limit
BEAM STRESSES		
Release		
Tension	none	0.200 ksi
Compression	3.068 ksi	4.2 ksi
Effective prestress, dead loads		
Tension	none	0.268 ksi
Compression	2.323 ksi	3.6 ksi
Effective prestress, dead loads, live load		
Tension	0.260 ksi	0.268 ksi
Compression	2.093 ksi	4.8 ksi
Live load and half the sum of effective prestress and dead loads		
Compression	1.332 ksi	3.2 ksi
DECK SLAB STRESS		
Composite dead loads		
Compression	0.115 ksi	2.25 ksi
Composite dead loads, live load		
Compression	0.788 ksi	3.0 ksi

FATIGUE LIMIT STATE (5.5.3.1)

For fully prestressed concrete components there is no need to check fatigue when the tensile stress in the extreme fiber at the Service III Limit State after all losses meets the tensile stress limits. Since this is the case, fatigue does not need to be checked.

STRENGTH LIMIT STATE

The Strength Limit State includes checks on the nominal flexural resistance and the amount of prestressed and non-prestressed reinforcement. For practical design the rectangular stress distribution can be used.

NOMINAL FLEXURAL RESISTANCE – MIDSPAN

The beams do not have any non-prestressed tension reinforcement or compression reinforcement. The stress block factor is based on the compressive strength of the deck concrete, assumes that the neutral axis remains within the deck slab. Since this is a strength limit state check, the width of the compression flange, b, is not reduced.

d_p = the distance from the extreme compression fiber to the centroid of the prestressing tendons (in)

β_1 = the stress block factor

1. Calculate the factored moments for the Strength I Limit State, M_u

M_u = (1.25)(2034 + 2053 + 250) + (1.5)(405) + (1.75)(3837) = 12744 k-ft (exterior beam)

M_u = (1.25)(2034 + 2137 + 250) + (1.5)(405) + (1.75)(2728) = 10908 k-ft (interior beam)

2. Calculate the depth of the compression block (5.7.3.1.1)

b = 114 inches

$d_p = e + y_t + t_{slab}$ = 32.09 + 35.62 + 9 = 76.71 in

k = 0.28 for low relaxation strand

β_1 = 0.80 for the 5 ksi deck concrete

In order to calculate the depth of the compression block, a, first calculate the depth to the neutral axis, c, assuming rectangular section behavior.

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A_s' f_y'}{0.85 f_c' \beta_1 b + k A_{ps} \frac{f_{pu}}{d_p}} = \frac{(9.114)(270) + 0 - 0}{(0.85)(5)(0.80)(114) + (0.28)(9.114)\left(\frac{270}{76.71}\right)} = 6.20 \text{ in}$$

Since the depth to the neutral axis is less than the slab thickness, the assumed rectangular section behavior is correct. The depth of the compression block is:

$a = \beta_1 c$ = (0.80)(6.20) = 4.96 in

3. Calculate the stress in the prestressing steel at the nominal flexural resistance (5.7.3.1.1)

For components with bonded tendons:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) = (270)\left(1 - (0.28)\left(\frac{6.20}{76.71}\right)\right) = 263.89 \text{ ksi}$$

4. Calculate the factored flexural resistance (5.7.3.2.1)

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) = (9.114)(263.89)\left(76.71 - \frac{4.96}{2}\right)\left(\frac{1}{12}\right) = 14878 \text{ k-ft}$$

$M_r = \varphi M_n$ = (1.0)(14878) = 14878 k-ft > 12744 k-ft for the exterior beam O.K.

> 10908 k-ft for the interior beam O.K.

REINFORCEMENT LIMITS – MIDSPAN

Check the reinforcement limits, maximum and minimum. The maximum amount of prestressed and non-prestressed reinforcement (5.7.3.3.1) should satisfy:

$$\frac{c}{d_e} \le 0.42$$

c = the distance from the extreme compression fiber to the neutral axis (in)

d_e = the corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in)

d_s = the distance from the extreme compression fiber to the centroid of the non-prestressed tensile reinforcement (in)

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = \frac{(9.114)(263.89)(76.71) + 0}{(9.114)(263.89) + 0} = 76.71 \text{ in}$$

$$\frac{c}{d_e} = \frac{6.20}{76.71} = 0.08 < 0.42 \quad \text{O.K.}$$

The minimum amount of prestressed and non-prestressed reinforcement is the amount needed to develop a factored flexural resistance, M_r, equal to the lesser of the following (5.7.3.3.2):

1.2 M_{cr}

1.33 times the factored moments (Strength Limit State, M_u)

The cracking moment, M_{cr}, may be taken as:

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \le S_c f_r$$

f_r = the modulus of rupture (ksi)

f_{cpe} = the compressive stress in the concrete due to the effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads (ksi)

S_c = the section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in^3)

S_{nc} = the section modulus for the extreme fiber of the monolithic or non-composite section where tensile stress is caused by externally applied loads (in^3)

M_{dnc} = the total unfactored dead load moment acting on the monolithic or non-composite section (k-ft)

M_u = the factored moment, Strength Limit State (k-ft)

Exterior Beam

M_{dnc} = 2034 + 2053 = 4087 k-ft = 49044 k-in

$$f_{cpe} = \frac{1505.2}{1085} - \frac{(1505.2)(-32.09)}{20157} = 3.7836 \text{ ksi}$$

$$M_{cr} = \left[(27751)(0.6788 + 3.7836) - (49044)\left(\frac{27751}{20157} - 1\right)\right]\left(\frac{1}{12}\right) = 8780 \text{ k-ft}$$

$S_c f_r$ = (27751)(0.6788) = 18837 k-in = 1570 k-ft

Since M_{cr} is to be less than or equal to $S_c f_r$,

1.2 M_{cr} = (1.2)(1570) = 1884 k-ft

1.33 M_u = (1.33)(12744) = 16950 k-ft

The lesser of these two values is 1884 k-ft and the factored flexural resistance is 17382 k-ft.

17382 k-ft > 1884 k-ft O.K.

Interior Beam

M_{dnc} = 2034 + 2137 = 4171 k-ft = 50052 k-in

$$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{8} = 0.6788 \text{ ksi} \quad (5.4.2.6)$$

$$f_{cpe} = \frac{P_e}{A} - \frac{P_e e}{S_{nc}} = \frac{1504.3}{1085} - \frac{(1504.3)(-33.09)}{20157} = 3.7813 \text{ ksi}$$

$$M_{cr} = \left[(27751)(0.6788+3.7813)-(50052)\left(\frac{27751}{20157}-1\right)\right]\left(\frac{1}{12}\right) = 8743 \text{ k-ft}$$

$S_c f_r$ = (27751)(0.6788) = 18837 k-in = 1570 k-ft

Since M_{cr} is to be less than or equal to $S_c f_r$,

1.2 M_{cr} = (1.2)(1570) = 1884 k-ft

1.33 M_u = (1.33)(10908) = 14508 k-ft

The lesser of these two values is 1884 k-ft and the factored flexural resistance is 17382 k-ft.

17382 k-ft > 1884 k-ft O.K.

PRETENSIONED ANCHORAGE ZONE (5.10.10.1)

The bursting resistance provided by vertical reinforcement in the ends of pretensioned beams at the service limit state should resist a force not less than 4% of the prestress force at transfer. The total reinforcement is located within a distance h/4 from the end of the beam. The stress in the steel, f_s, is not to exceed 20 ksi. Bursting resistance provided by the vertical reinforcement:

$P_r = f_s A_s$

A_s = the total area of vertical reinforcement located within the distance h/4 from the end of the beam (in^2)

h = the overall depth of the precast member

P_t = the prestressing force at transfer (k)

$P_r = (P_t)(0.04)$ = (1691.2)(0.04) = 67.65 k

h = 72 inches

$$\frac{h}{4} = \frac{72}{4} = 18.0 \text{ in}$$

Solve for the area of vertical reinforcement required:

$$A_s = \frac{P_r}{f_s} = \frac{67.45}{20} = 3.38 \text{ in}^2$$

Using pairs of No. 4 bars, A_s = 0.40 in^2 and the number of pairs of bars required is:

$$\frac{3.38}{0.40} = 8.5$$

With 9 pairs of No. 4 bars:

A_s = (9)(0.40) = 3.60 in^2

DEVELOPMENT AND TRANSFER LENGTH

The development length, l_d, in prestressing strand is the length of strand over which there is a gradual buildup of strand force, Figure 9. It consists of two components, the transfer and the flexural bond lengths. The prestress force varies linearly over the transfer length. At the end of the transfer length the stress in the strand is the effective prestress stress. The prestress force then varies in a parabolic manner and reaches the tensile strength of the strand at the end of the development length. The transfer and development lengths are calculated. The flexural bond length can be calculated by taking the difference between the development and transfer lengths. This example calculates the development lengths using the stress in the prestressing steel previously calculated at midspan. The transfer length only depends on the strand diameter and therefore is the same for the interior and exterior beams, sixty strand diameters. The development length for bonded strands:

$$\ell_d \geq k\left(f_{ps} - \frac{2}{3}f_{pe}\right)d_b$$

d_b = the nominal strand diameter (in)

f_{ps} = the average stress in the prestressing steel at the time for which the nominal resistance of the member is required (ksi)

k = 1.6 for precast prestressed beams

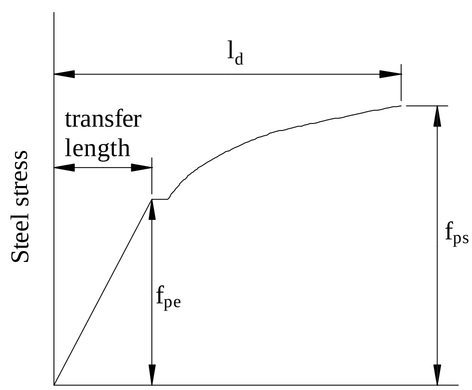

Figure 9 – Development Length

TRANSFER LENGTH (5.11.4.1)

$60\ d_b = (60)(0.6) = 36$ in

DEVELOPMENT LENGTH (5.11.4.2)

Exterior Beam

$$k\left(f_{ps} - \frac{2}{3}f_{pe}\right)d_b = (1.6)\left(263.89 - \left(\frac{2}{3}\right)(160.15)\right)(0.6) = 150.8 \text{ in}$$

Interior Beam

$$k\left(f_{ps} - \frac{2}{3}f_{pe}\right)d_b = (1.6)\left(263.89 - \left(\frac{2}{3}\right)(160.05)\right)(0.6) = 150.9 \text{ in}$$

WEB SHEAR DESIGN (5.8)

Determine the required transverse reinforcement spacing at the critical section and at the inside edge of the elastomeric bearing. Verify that the longitudinal reinforcement on the flexural tension side of the member is properly proportioned at these locations. The calculation of the critical section is an iterative process because the criteria used to calculate the location depends on the value of the effective shear depth at the location of the critical section.

V_c = the nominal shear resistance of the concrete (k)
V_p = the component in the direction of the applied shear of the effective prestressing force (k)
V_n = the nominal shear resistance of the section (k)
V_s = the nominal shear resistance of the shear reinforcing steel (k)
d_v = the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in)
s = the spacing of the shear reinforcement (in)
β = the factor indicating the ability of diagonally cracked concrete to transmit tension
θ = the angle of inclination of the diagonal compressive stresses (deg)
b_v = the effective web width, taken as the minimum web width within the depth d_v (in)
h = the overall depth of the member
A_v = the area of shear reinforcement within a distance s (in^2)
A_c = the area of concrete on the flexural tension side of the member (in^2)

Exterior Beam

1. Calculate the effective shear depth, d_v (5.8.2.9)

One way to calculate the effective shear depth is to calculate the depth of the compression block and then the distance between the middle of the compression block and the center of gravity of all the tensile reinforcement. Since non-prestressed reinforcement is not used, d_e is equal to d_p. Starting with an estimated location of the critical section at the 0.05 point:

$e = 14.56 + (18.95 - 14.56)(0.5) = 16.76$ in

$d_e = d_p = e + y_t + t_{slab} = 16.76 + 35.62 + 9 = 61.38$ in

Assuming rectangular section behavior:

$$c = \frac{A_{ps}f_{pu} + A_sf_y - A_s'f_y'}{0.85f_c'\beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{(9.114)(270)+0-0}{(0.85)(5)(0.80)(114) + (0.28)(9.114)\left(\frac{270}{61.38}\right)} = 6.17 \text{ in}$$

Since the depth to the neutral axis is less than the slab thickness, the assumed rectangular section behavior is correct. The depth of the compression block is:

$a = \beta_1 c = (0.80)(6.17) = 4.94$ in

The calculated effective shear depth is:

$$d_v = d_e - \frac{a}{2} = 61.38 - \frac{4.94}{2} = 58.91 \text{ in}$$

But d_v does not need to be less than the greater of:

$0.9d_e = (0.9)(61.38) = 55.24$ in

$0.72h = (0.72)(81) = 58.32$ in

Since the calculated value is greater than both of these values, $d_v = 58.91$ in

2. Calculate the location of the critical section (5.8.3.2)

The critical section is located a distance d_v from the face of the support. Since the location of the critical section is measured from the face of the support, add a distance equal to one-half the width of the bearing pad or 4 inches. Therefore, the distance from the center of bearing to the critical section is:

58.91 + 4.00 = 62.91 in (~ 0.044 point)

Since the revised location of the critical section is not close enough to the previous value, repeat.

3. Calculate the effective shear depth, d_v (5.8.2.9)

At the revised estimated location of the critical section, the 0.044 point:

e = 14.56 + (18.95 – 14.56)(0.44) = 16.49 in

$d_e = d_p = e + y_t + t_{slab} = 16.49 + 35.62 + 9 = 61.11$ in

Assuming rectangular section behavior:

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{(9.114)(270)+0-0}{(0.85)(5)(0.80)(114) + (0.28)(9.114)\left(\frac{270}{61.11}\right)} = 6.17 \text{ in}$$

Since the depth to the neutral axis is less than the slab thickness, the assumed rectangular section behavior is correct. The depth of the compression block is:

$a = \beta_1 c = (0.80)(6.17) = 4.94$ in

The calculated effective shear depth is:

$$d_v = d_e - \frac{a}{2} = 61.11 - \frac{4.94}{2} = 58.64 \text{ in}$$

But d_v does not need to be less than the greater of:

$0.9d_e = (0.9)(61.11) = 55.00$ in

$0.72h = 58.32$ in

Since the calculated value is greater than both of these values, $d_v = 58.64$ in

4. Calculate the location of the critical section (5.8.3.2)

The critical section is located a distance d_v from the face of the support. Therefore, the distance from the center of bearing to the critical section is:

58.64 + 4.00 = 62.64 in (~ 0.044 point)

Since the revised location of the critical section is close enough to the previous value, continue with the next step.

5. Calculate the factored loads for the Strength I Limit State

Table 6 – Beam Moments and Shears, 0.044 Point

Load	Dead Load				Live Load plus Dynamic Load Allowance
	DC			DW	
	Beam	Slab/Diaph	Rail	FWS	
Moment, k-ft	342	339	42	68	663
Shear, k	61.8	61.3	7.6	12.3	125.4

$M_u = (1.25)(342 + 339 + 42) + (1.5)(68) + (1.75)(663) = 2166$ k-ft

$V_u = (1.25)(61.8 + 61.3 + 7.6) + (1.5)(12.3) + (1.75)(125.4) = 401.3$ k

$N_u = 0$ k

6. Calculate the prestress tendon shear component

ψ = the inclination of the harped strands (deg)

$$\psi = \tan^{-1}\left(\frac{62}{582}\right) = 6.08^{o}$$

$V_p = (A_{ps})(f_{pe})(\sin\psi) = (12)(0.217)(160.15)(0.1059) = 44.16$ k

Since the critical section is located past the transfer length, use the full effective stress.

7. Calculate the shear stress in the concrete and the shear stress ratio (5.8.2.9)

$b_v = 8$ inches

The shear stress in the concrete is:

$$v_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{401.3 - (0.9)(44.16)}{(0.9)(8)(58.64)} = 0.856 \text{ ksi}$$

The shear stress ratio is:

$$\frac{v_u}{f_c^{'}} = \frac{0.856}{8} = 0.107$$

8. Calculate the strain in reinforcement on the flexural tension side of the member (5.8.3.4.2)

A_{ps} = the area of prestressing steel on the flexural tension side of the member (in^2)

A_s = the area of non-prestressed steel on the flexural tension side of the member at the section under investigation (in^2)

f_{po} = a parameter taken as the modulus of elasticity of prestressing tendons multiplied by the locked in difference in strain between the prestressing tendons and the surrounding concrete, for usual levels of prestressing, 0.70 f_{pu} will be appropriate

M_u = the factored moment not to be taken as less than $V_u d_v$

$A_{ps} = (0.217)(30) = 6.510$ in^2 (30 strands)

$A_s = 0$ in^2

$A_c = 584$ in^2

$f_{po} = (0.7)(270) = 189$ ksi

$V_u d_v = (401.3)(58.64) = 23532$ k-in = 1961 k-ft

Since this value is less than the actual applied factored moment, use the actual applied factored moment. Assume an initial value for θ of 23.7°, which is based on the shear stress ratio of 0.107 and an assumed value for ε_x of zero. The initial value of ε_x should not be taken greater than 0.001.

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5\left(V_u - V_p\right)\cot\theta - A_{ps}f_{po}}{2\left(E_s A_s + E_{ps}A_{ps}\right)}$$

$$\varepsilon_x = \frac{\frac{(2166)(12)}{58.64} + 0 + (0.5)(401.3 - 44.16)(\cot 23.7) - (6.510)(189)}{(2)\left[0 + (28500)(6.510)\right]} = -1.0250 \times 10^{-3} \text{ in / in}$$

Since ε_x is negative:

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5\left(V_u - V_p\right)\cot\theta - A_{ps}f_{po}}{2\left(E_cA_c + E_sA_s + E_{ps}A_{ps}\right)}$$

$$\varepsilon_x = \frac{\frac{(2166)(12)}{58.64} + 0 + (0.5)(401.3 - 44.16)(\cot 23.7) - (6.510)(189)}{(2)\left[(5154)(584) + 0 + (28500)(6.510)\right]} = -\,0.0595 \text{ x } 10^{-3} \text{ in / in}$$

Use the shear stress ratio and ε_x to obtain θ from Table 5.8.3.4.2-1, 22.8˚. Since this does not agree with the initial value, repeat.

$$\varepsilon_x = \frac{\frac{(2166)(12)}{58.64} + 0 + (0.5)(401.3 - 44.16)(\cot 22.8) - (6.510)(189)}{(2)\left[0 + (28500)(6.510)\right]} = -\,0.9765 \text{ x } 10^{-3} \text{ in / in}$$

Since the revised ε_x is negative:

$$\varepsilon_x = \frac{\frac{(2166)(12)}{58.64} + 0 + (0.5)(401.3 - 44.16)(\cot 22.8) - (6.510)(189)}{(2)\left[(5154)(584) + 0 + (28500)(6.510)\right]} = -\,0.0567 \text{ x } 10^{-3} \text{ in / in}$$

Use the shear stress ratio and ε_x to obtain θ, 22.8˚. Since this agrees with the previous value, continue.

9. Obtain the final theta and beta factors from Table 5.8.3.4.2-1

$\theta = 22.8$

$\beta = 2.94$

10. Calculate the shear resistance provided by the concrete, V_c (5.8.3.3)

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v = (0.0316)(2.94)\left(\sqrt{8}\right)(8)(58.64) = 123.27 \text{ k}$$

11. Check if the region requires transverse reinforcement (5.8.2.4)

Transverse reinforcement is required if:

$V_u > 0.5\varphi(V_c + V_p)$

$0.5\varphi(V_c + V_p) = (0.5)(0.9)(123.27 + 44.16) = 75.34$ k

Since V_u, 401.3 k, is greater than this value, transverse reinforcement is required

12. Calculate the shear resistance that the transverse reinforcement needs to provide, V_s

$$V_s = \frac{V_u}{\varphi} - V_c - V_p = \frac{401.3}{0.9} - 123.27 - 44.16 = 278.46 \text{ k}$$

13. Calculate the required transverse reinforcement spacing

$A_v = 0.40$ in^2 (2 No. 4 bars)

$$s = \frac{A_v f_y d_v \cot\theta}{V_s} = \frac{(0.40)(60)(58.64)(\cot\ 22.8)}{278.46} = 12.0 \text{ in}$$

Based on reinforcement placed at 90˚ to the longitudinal axis of the beam

14. Calculate the maximum allowable spacing of the transverse reinforcement (5.8.2.7)

$$0.125f'_c = (0.125)(8) = 1.000 \text{ ksi}$$

Since $v_u = 0.856 \text{ ksi} < 0.125f'_c$

$s_{max} \le 0.8d_v \le 24.0 \text{ in}$

$0.8d_v = (0.8)(58.64) = 46.9 \text{ in} > 24.0 \text{ in}$

Therefore, the maximum spacing is 24 inches. Use 2 No. 4 bars at a 12-inch spacing.

15. Check if the minimum amount of transverse reinforcement is provided (5.8.2.5)

$$A_v = 0.0316\sqrt{f'_c}\frac{b_v s}{f_y} = (0.0316)(\sqrt{8})\left(\frac{(8)(12)}{60}\right) = 0.14 \text{in}^2$$

Since the transverse reinforcement provided, 0.40 in^2, is greater than this, the minimum transverse reinforcement provision is satisfied.

16. Check the nominal shear resistance (5.8.3.3)

The nominal shear resistance is the lesser of:

$$V_n \le 0.25f'_c b_v d_v + V_p$$

$$V_c + V_s + V_p$$

$$V_s = \frac{A_v f_y d_v \cot\theta}{s} = \frac{(0.40)(60)(58.64)(\cot\ 22.8)}{12} = 279.00 \text{ k}$$

$$0.25f'_c b_v d_v + V_p = (0.25)(8)(8)(58.64) + 44.16 = 982.40 \text{ k}$$

$V_c + V_s + V_p = 123.27 + 279.00 + 44.16 = 446.43 \text{ k}$

The lesser of these two values is 446.43 k and the factored shear resistance is:

$V_r = \varphi V_n = (0.9)(446.43) = 401.79 \text{ k} > 401.3 \text{ k}$ O.K.

17. Check the longitudinal reinforcement requirement (5.8.3.5)

At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_s f_y + A_{ps} f_{ps} \ge \frac{M_u}{d_v \varphi} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta$$

Any lack of full development shall be accounted for.

A_{ps} = the area of prestressing steel on the flexural tension side of the member (in^2)

A_s = the area on non-prestressed steel on the flexural tension side of the member (in^2)

V_s = the shear resistance provided by the transverse reinforcement at the section under investigation, given by Equation 5.8.3.3-4, except V_s shall not be taken as greater than $\frac{V_u}{\varphi}$

Consider the lack of full development of the prestressing steel using the plot of the stress in the prestressing steel versus the distance from the end of the strand. At the end of the transfer length, 36 inches, the stress in the prestressing strand is f_{pe}, 160.2 ksi. At the end of the development length, 151 inches, the stress in the prestressing strand, f_{ps}, is 263.9 ksi. Using a straight-line interpolation between the end of the transfer length and the development length the stress in the prestressing strand at 68 inches from the end of the beam is approximately:

$$f_{ps} \approx 160.2 + \left(\frac{263.9 - 160.2}{151 - 36}\right)(68 - 36) = 189 \text{ ksi}$$

Calculate the tensile capacity provided by the longitudinal reinforcement:

$A_s f_y + A_{ps} f_{ps} = 0 + (6.510)(189) = 1230$ k

Calculate the required tensile force where V_s is not to be greater than:

$$\frac{V_u}{\varphi} = \frac{401.3}{0.9} = 445.89 \text{ k}$$

Since the nominal shear resistance provide, 279.00 k, is less than this value, use the nominal shear resistance provided.

$$A_s f_y + A_{ps} f_{ps} \geq \frac{(2166)(12)}{(58.64)(1.0)} + 0 + \left(\frac{401.3}{0.9} - (0.5)(279.00) - 44.16\right)(\cot 22.8) = 1067 \text{ k}$$

Since 1230 k > 1067 k O.K.

Interior Beam

1. Calculate the effective shear depth, d_v (5.8.2.9)

Starting with an estimated location of the critical section at the 0.05 point:

e = 16.76 in

$d_e = d_p = 61.38$ in

Assuming rectangular section behavior:

$$c = \frac{(9.114)(270) + 0 - 0}{(0.85)(5)(0.80)(114) + (0.28)(9.114)\left(\frac{270}{61.38}\right)} = 6.17 \text{ in}$$

Since the depth to the neutral axis is less than the slab thickness, the assumed rectangular section behavior is correct. The depth of the compression block is:

$a = \beta_1 c = (0.80)(6.17) = 4.94$ in

The calculated effective depth is:

$$d_v = 61.38 - \frac{4.94}{2} = 58.91 \text{ in}$$

But d_v does not need to be less than the greater of:

$0.9d_e = (0.9)(61.38) = 55.24$ in

0.72h = 58.32 in

Since the calculated value is greater than both of these values, $d_v = 58.91$ in

2. Calculate the location of the critical section (5.8.3.2)

The critical section is located a distance d_v from the face of the support. Therefore, the distance from the center of bearing to the critical section is:

58.91 + 4.00 = 62.91 in (~ 0.044 point)

Since the revised location of the critical section is not close enough to the previous value, repeat.

3. Calculate the effective shear depth, d_v (5.8.2.9)

At the revised estimated location of the critical section, the 0.044 point:

e = 14.56 + (18.95 – 14.56)(0.44) = 16.49 in

$d_e = d_p = e + y_t + t_{slab} = 16.49 + 35.62 + 9 = 61.11$ in

Assuming rectangular section behavior:

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{(9.114)(270) + 0 - 0}{(0.85)(5)(0.80)(114) + (0.28)(9.114)\left(\frac{270}{61.11}\right)} = 6.17 \text{ in}$$

Since the depth to the neutral axis is less than the slab thickness, the assumed rectangular section behavior is correct. The depth of the compression block is:

$a = \beta_1 c = (0.80)(6.17) = 4.94$ in

The calculated effective shear depth is:

$$d_v = d_e - \frac{a}{2} = 61.11 - \frac{4.94}{2} = 58.64 \text{ in}$$

But d_v does not need to be less than the greater of:

$0.9d_e = (0.9)(61.11) = 55.00$ in

$0.72h = 58.32$ in

Since the calculated value is greater than both of these values, $d_v = 58.64$ in

4. Calculate the location of the critical section (5.8.3.2)

The critical section is located a distance d_v from the face of the support. Therefore, the distance from the center of bearing to the critical section is:

58.64 + 4.00 = 62.64 in (~ 0.044 point)

Since the revised location of the critical section is close enough to the previous value, continue with the next step.

5. Calculate the factored loads for the Strength I Limit State

Table 7 – Beam Moments and Shears, 0.044 Point					
Load	Dead Load				Live Load plus Dynamic Load Allowance
	DC			DW	
	Beam	Slab/Diaph	Rail	FWS	
Moment, k-ft	342	346	42	68	472
Shear, k	61.8	62.7	7.6	12.3	110.5

$M_u = (1.25)(342 + 346 + 42) + (1.5)(68) + (1.75)(472) = 1841$ k-ft

$V_u = (1.25)(61.8 + 62.7 + 7.6) + (1.5)(12.3) + (1.75)(110.5) = 377.0$ k

$N_u = 0$ k

6. Calculate the prestress tendon shear component

$V_p = (A_{ps})(f_{pe})(\sin\psi) = (12)(0.217)(160.05)(0.1059) = 44.14$ k

Since the critical section is located past the transfer length, use the full effective stress.

7. Calculate the shear stress in the concrete and the shear stress ratio (5.8.2.9)

The shear stress in the concrete is:

$$v_u = \frac{377.0 - (0.9)(44.14)}{(0.9)(8)(58.64)} = 0.799 \text{ ksi}$$

The shear stress ratio is:

$$\frac{v_u}{f_c^{'}} = \frac{0.799}{8} = 0.100$$

8. Calculate the strain reinforcement on the flexural tension side of the member (5.8.3.4.2)

$V_u d_v$ = (377.0)(58.64) = 22107 k-in = 1842 k-ft

Since this value is greater than the actual applied factored moment, use this value.

Assume an initial value for θ of 22.5˚, which is based on the shear stress ratio of 0.100 and an assumed value for ε_x of zero.

$$\varepsilon_x = \frac{\frac{(1842)(12)}{58.64} + (0.5)(377.0 - 44.14)(\cot 22.5) - (6.510)(189)}{(2)\left[0 + (28500)(6.510)\right]} = -1.2172 \text{ x } 10^{-3} \text{ in / in}$$

Since ε_x is negative:

$$\varepsilon_x = \frac{\frac{(1842)(12)}{58.64} + 0 + (0.5)(377.0 - 44.14)(\cot 22.5) - (6.510)(189)}{(2)\left[(5154)(584) + 0 + (28500)(6.510)\right]} = -0.0707 \text{ x } 10^{-3} \text{ in / in}$$

Use the shear stress ratio and ε_x to obtain θ from Table 5.8.3.4.2-1, 210.4˚. Since this does not agree with the initial value, repeat.

$$\varepsilon_x = \frac{\frac{(1842)(12)}{58.64} + (0.5)(377.0 - 44.14)(\cot 21.4) - (6.510)(189)}{(2)\left[0 + (28500)(6.510)\right]} = -1.1555 \text{ x } 10^{-3} \text{ in / in}$$

Since the revised ε_x is negative:

$$\varepsilon_x = \frac{\frac{(1842)(12)}{58.64} + 0 + (0.5)(377.0 - 44.14)(\cot 21.4) - (6.510)(189)}{(2)\left[(5154)(584) + 0 + (28500)(6.510)\right]} = -0.0671 \text{ x } 10^{-3} \text{ in / in}$$

Use the shear stress ratio and the ε_x to obtain θ, 21.4˚. Since this agrees with the previous value, continue.

9. Obtain the final theta and beta factors from Table 5.8.3.4.2-1

θ = 21.4

β = 3.24

10. Calculate the shear resistance provided by the concrete, V_c (5.8.3.3)

$$V_c = (0.0316)(3.24)\left(\sqrt{8}\right)(8)(58.64) = 135.85 \text{k}$$

11. Check if the region requires transverse reinforcement (5.8.2.4)

$0.5\varphi(V_c + V_p)$ = (0.5)(0.9)(135.85 + 44.14) = 81.00 k

Since V_u, 377.0 k, is greater than this value, transverse reinforcement is required.

12. Calculate the shear resistance that the transverse reinforcement needs to provide, V_s

$$V_s = \frac{377.0}{0.9} - 135.85 - 44.14 = 238.90\ \text{k}$$

13. Calculate the required transverse reinforcement spacing

$$s = \frac{(0.40)(60)(58.64)(\cot 21.4)}{238.90} = 15.0\ \text{in}$$

14. Calculate the maximum allowable spacing of the transverse reinforcement (5.8.2.7)

$$0.125f'_c = (0.125)(8) = 1.000\ \text{ksi}$$

Since $v_u = 0.799\ \text{ksi} < 0.125f'_c$

$s_{max} \leq 0.8d_v \leq 24.0\ \text{in}$

$0.8d_v = (0.8)(58.64) = 46.9\ \text{in} > 24.0\ \text{in}$

Therefore, the maximum spacing is 24 inches. Use 2 No. 4 bars at the 12-inch spacing used in the exterior beam.

13. Check if the minimum amount of transverse reinforcement is provided (5.8.2.5)

Since using the same transverse reinforcement spacing as the exterior beam, the minimum transverse reinforcement provision is satisfied.

14. Check the nominal shear resistance (5.8.3.3)

$$V_s = \frac{(0.40)(60)(58.64)(\cot 21.4)}{12} = 299.26\ \text{k}$$

$$0.25f'_c b_v d_v + V_p = (0.25)(8)(8)(58.64) + 44.14 = 982.38\ \text{k}$$

$V_c + V_s + V_p = 135.85 + 299.26 + 44.14 = 479.25\ \text{k}$

The lesser of these two values is 479.25 k and the factored shear resistance is:

$V_r = \varphi V_n = (0.9)(479.25) = 431.33\ \text{k} > 377.0\ \text{k}$ O.K.

15. Check the longitudinal reinforcement requirement (5.8.3.5)

Consider the lack of full development of the prestressing steel using the plot of the stress in the prestressing steel versus the distance from the end of the strand. At the end of the transfer length, 36 inches, the stress in the prestressing strand is f_{pe}, 160.1 ksi. At the end of the development length, 151 inches, the stress in the prestressing strand, f_{ps}, is 263.9 ksi. Using a straight-line interpolation between the end of the transfer length and the development length the stress in the prestressing strand at 68 inches from the end of the beam is approximately:

$$f_{ps} \approx 160.1 + \left(\frac{263.9 - 160.1}{151 - 36}\right)(68 - 36) = 189\ \text{ksi}$$

Calculate the tensile capacity provided by the longitudinal reinforcement:

$A_s f_y + A_{ps} f_{ps} = 0 + (6.510)(189) = 1230\ \text{k}$

Calculate the required tensile force where V_s is not to be greater than:

$$\frac{V_u}{\varphi} = \frac{377.0}{0.9} = 418.89\ \text{k}$$

Since the nominal shear resistance provided, 299.26 k, is less than this value, use the nominal shear resistance provided.

$$A_s f_y + A_{ps} f_{ps} \geq \frac{(1841)(12)}{(58.64)(1.0)} + 0 + \left(\frac{377.0}{0.9} - (0.5)(299.26) - 44.14\right)(\cot 21.4) = 951 \text{ k}$$

Since 1230 k > 951 k O.K.

At the inside edge of the bearing area of simple end supports to the section of critical shear, the longitudinal reinforcement on the flexural tension side of the member shall satisfy:

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta$$

The inside edge of the elastomeric bearing is the sum of the distance from the end of the beam to the centerline of bearing and one-half of the bearing pad.

Table 8 – Beam Shears, Centerline of Bearing (k)					
Beam	Dead Load				Live Load plus Dynamic Load Allowance
	DC			DW	
	Beam	Slab/Diaph	Rail	FWS	
Interior	67.8	68.4	8.3	13.5	117.4
Exterior	67.8	67.0	8.3	13.5	133.2

Exterior Beam – inside edge of bearing area

1. Calculate the effective shear depth, d_v (5.8.2.9)

Using the values from at the centerline of bearing:

$d_e = d_p = 14.56 + 35.62 + 9 = 59.18$ in

Assuming rectangular behavior:

$$c = \frac{(9.114)(270) + 0 - 0}{(0.85)(5)(0.80)(114) + (0.28)(9.114)\left(\frac{270}{59.18}\right)} = 6.16 \text{ in}$$

Since the depth to the neutral axis is less than the slab thickness, the assumed rectangular section behavior is correct. The depth of the compression block is:

a = (0.80)(6.16) = 4.93 in

The calculated shear depth is:

$$d_v = 59.18 - \frac{4.93}{2} = 56.72 \text{ in}$$

But d_v does not need to be less than the greater of:

$0.9d_e = (0.9)(59.18) = 53.26$ in

$0.72h = 58.32$ in

Since the calculated value is less than one of these values, $d_v = 58.32$ in

2. Calculate the factored shear for the Strength I Limit State, V_u

Using the shears from at the centerline of bearing:

$V_u = (1.25)(67.8 + 67.0 + 8.3) + (1.5)(13.5) + (1.75)(133.2) = 432.2$ k

3. Calculate the prestress tendon shear component

$$f_{ps} = \left(\frac{10}{36}\right)(160.15) = 44.49 \text{ ksi}$$

$V_p = (A_{ps})(f_{ps})(\sin\psi) = (12)(0.217)(44.49)(0.1059) = 12.27$ k

Since the inside edge of the bearing area is located within the transfer length, the stress in the prestressing strands is a proportion of the effective prestressing stress.

4. Calculate the shear stress in the concrete and the shear stress ratio (5.8.2.9)

The shear stress in the concrete is:

$$v_u = \frac{432.2 - (0.9)(12.27)}{(0.9)(8)(58.32)} = 1.003 \text{ ksi}$$

The shear stress ratio is:

$$\frac{v_u}{f_c'} = \frac{1.003}{8} = 0.125$$

5. Calculate the strain in reinforcement on the flexural tension side of the member (5.8.3.4.2)

$$f_{po} = (0.7)(270)\left(\frac{10}{36}\right) = 52.50 \text{ ksi}$$

$V_u d_v = (432.2)(58.32) = 25206$ k-in $= 2100$ k-ft

Since this value is greater than the actual applied factored moment, use this value. Also, since the inside edge of the bearing area is located within the transfer length, the value of f_{po} is a proportion of the full value of 189 ksi. Assume an initial value for θ of 23.7˚, which is based on the shear stress ratio of 0.125 and an assumed value for ε_x of zero.

$$\varepsilon_x = \frac{\frac{(2100)(12)}{58.32} + 0 + (0.5)(432.2 - 12.27)(\cot 23.7) - (6.510)(52.50)}{(2)\left[0 + (28500)(6.510)\right]} = 1.5325 \text{ x } 10^{-3} \text{ in / in}$$

Use the shear stress ratio and ε_x to obtain θ from Table 5.8.3.4.2-1, 37.0˚. Since this does not agree with the initial value, repeat.

$$\varepsilon_x = \frac{\frac{(2100)(12)}{58.32} + 0 + (0.5)(432.2 - 12.27)(\cot 37.0) - (6.510)(52.50)}{(2)\left[0 + (28500)(6.510)\right]} = 0.9943 \text{ x } 10^{-3} \text{ in / in}$$

Use the shear stress ratio and ε_x to obtain θ, 37.0˚. Since this agrees with the previous value, continue.

6. Calculate the shear resistance provided by the transverse reinforcement, V_s (5.8.3.3)

Using the transverse reinforcement spacing previously calculated, 12 inches:

$$V_s = \frac{(0.40)(60)(58.32)(\cot 37.0)}{12} = 154.79 \text{ ksi}$$

7. Check the longitudinal reinforcement requirement (5.8.3.5)

Calculate the tensile capacity of the longitudinal reinforcement:

$A_s f_y + A_{ps} f_{ps} = 0 + (6.510)(44.49) = 290$ k

Calculate the required tensile force where V_s is not to be greater than:

$$\frac{V_u}{\varphi} = \frac{432.2}{0.9} = 480.22 \text{ k}$$

Since the nominal shear resistance provided, 154.79 k, is less than this value, use the nominal shear resistance provided.

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta = \left(\frac{432.2}{0.9} - (0.5)(154.79) - 12.27\right)(\cot 37.0) = 518 \text{ k}$$

However, 290 k < 518 k N.G.

Try decreasing the transverse reinforcement spacing so that the maximum value allowed for V_s, 480.22 k, controls. The required tensile force is now:

$$A_s f_y + A_{ps} f_{ps} = \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta = \left(\frac{432.2}{0.9} - (0.5)(480.22) - 12.27\right)(\cot 37.0) = 302 \text{ k}$$

However, 290 k < 302 k N.G.

Add non-prestressed reinforcement, No. 5 bars

The basic development length for No. 5 bars is:

$$\frac{1.25A_b f_y}{\sqrt{f_c'}}$$

But is not less than:

$$0.4 d_b f_y$$

$$\frac{1.25A_b f_y}{\sqrt{f_c'}} = \frac{(1.25)(0.31)(60)}{\sqrt{8}} = 8.22 \text{ in}$$

$0.4d_bf_y = (0.4)(0.625)(60) = 15$ in

The larger of these two values is 15 inches and therefore, the development length is 15 inches. Since the inside edge of the bearing area is located within the development length, 10 inches from the end of the beam, consider the lack of full development. The tensile capacity provided by the longitudinal reinforcement is now:

$$A_s f_y + A_{ps} f_{ps} = \left(\frac{10}{15}\right)(A_s)(60) + (6.510)(44.49) = 302 \text{ k}$$

Solving for the area of reinforcement required:

$A_s = 0.31$ in^2

Add 2 No. 5 bars, $A_s = 0.62$ in^2

$$A_s f_y + A_{ps} f_{ps} = \left(\frac{10}{15}\right)(0.62)(60) + (6.510)(44.49) = 314 \text{ k}$$

Since 314 k > 302 k O.K.

Calculate the revised transverse reinforcement spacing.

$$s = \frac{(0.40)(60)(58.32)(\cot 37.0)}{480.22} = 3.87 \text{ in}$$

Use 3.75-inch transverse reinforcement spacing.

Interior Beam – inside edge of bearing area

1. Calculate the effective shear depth, d_v (5.8.2.9)

   The effective shear depth for the interior beam is the same as the exterior beam, same strand pattern and section properties, 58.32 inches.

2. Calculate the factored shear for the Strength I Limit State, V_u

   Using the shears from at the centerline of bearing:

   $V_u = (1.25)(67.8 + 68.4 + 8.3) + (1.5)(13.5) + (1.75)(117.4) = 406.3$ k

3. Calculate the prestress tendon shear component

$$f_{ps} = \left(\frac{10}{36}\right)(160.05) = 44.46 \text{ ksi}$$

   $V_p = (A_{ps})(f_{ps})(\sin\psi) = (12)(0.217)(44.46)(0.1059) = 12.26$ k

   Once again the stress in the prestressing strands is reduced because the inside edge of the bearing area is located within the transfer length.

4. Calculate the shear stress in the concrete and the shear stress ratio (5.8.2.9)

   The shear stress in the concrete is:

$$v_u = \frac{406.3 - (0.9)(12.26)}{(0.9)(8)(58.32)} = 0.941 \text{ ksi}$$

   The shear stress ratio is:

$$\frac{v_u}{f'_c} = \frac{0.941}{8} = 0.118$$

5. Calculate the strain in reinforcement on the flexural tension side of the member (5.8.3.4.2)

   $V_u d_v = (406.3)(58.32) = 23695$ k-in $= 1975$ k-ft

   Since this value is greater than the actual applied factored moment, use this value. Once again the value of f_{po} is reduced because the inside edge of the bearing area is located within the transfer length. Assume an initial value for θ of 23.7˚, which is based on the shear stress ratio of 0.118 and an assumed value for ε_x of zero.

$$\varepsilon_x = \frac{\frac{(1975)(12)}{58.32} + 0 + (0.5)(406.3 - 12.26)(\cot 23.7) - (6.510)(52.50)}{(2)\left[0 + (28500)(6.510)\right]} = 1.3836 \times 10^{-3} \text{ in / in}$$

   Use the shear stress ratio and ε_x to obtain θ, 37.0˚. Since this does not agree with the initial value, repeat.

$$\varepsilon_x = \frac{\frac{(1975)(12)}{58.32} + 0 + (0.5)(406.3 - 12.26)(\cot 37.0) - (6.510)(52.50)}{(2)\left[0 + (28500)(6.510)\right]} = 0.8787 \times 10^{-3} \text{ in / in}$$

   Use the shear stress ratio and ε_x to obtain θ, 37.0˚. Since this agrees with the previous value, continue.

6. Calculate the shear resistance provided by the transverse reinforcement, V_s (5.8.3.3)

   Using the transverse reinforcement spacing calculated for the interior beam, 3.75 inches:

$$V_s = \frac{(0.40)(60)(58.32)(\cot 37.0)}{3.75} = 495.32 \text{ k}$$

7. Check the longitudinal reinforcement requirement (5.8.3.5)

Calculate the tensile capacity provided by the longitudinal reinforcement:

$$A_s f_y + A_{ps} f_{ps} = \left(\frac{10}{15}\right)(0.62)(60) + (6.510)(44.46) = 314 \text{ k}$$

Calculate the required tensile force where V_s is not to be greater than:

$$\frac{V_u}{\varphi} = \frac{406.3}{0.9} = 451.44 \text{ k}$$

Since the nominal shear resistance provided, 495.32 k, is greater than this value, use the limit on nominal shear resistance.

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta = \left(\frac{406.3}{0.9} - (0.5)(451.44) - 12.26\right)(\cot 37.0) = 283 \text{ k}$$

Since 314 k > 283 k O.K.

INTERFACE SHEAR TRANSFER (5.8.4)

For composite prestressed concrete beams, interface shear transfer is considered across the plane at the interface between two concrete cast at different times. The cross-section area, A_{vf}, of the reinforcement crossing the shear plane is calculated per unit length along the beam or girder. The maximum longitudinal spacing of the rows of reinforcing bars is 24 inches. If the width of the contact surface exceeds 48.0 inches, a minimum of four bars per rows should be used (commentary provision). Evaluate interface shear transfer at the critical section.

A_{cv} = the area of concrete engaged in shear transfer (in^2)
A_{vf} = the area of shear reinforcement crossing the shear plane (in^2)
b_v = the width of the interface (in)
c = the cohesion factor (ksi)
μ = the friction factor
P_c = the permanent net compressive force normal to the shear plane (k)
d_e = the distance between the centroid of the steel on the tension side of the beam to the center of the compression block in the deck (in)
f'_c = the specified 28-day compressive strength of the weaker concrete (ksi)

Exterior Beam – 0.044 Point

1. Calculate the factored vertical shear force for the Strength I Limit State, composite loads, V_u

V_u = (1.25)(7.6) + (1.5)(12.3) + (1.75)(125.4) = 247.4 k

2. Calculate the distance, d_e

The distance from the bottom of the beam to the centroid of the steel on the tension side of the beam is:

$$\bar{y} = \frac{(10)(2) + (10)(4) + (10)(6)}{30} = 4.00 \text{ in}$$

The distance from the top of the deck slab to the center of the compression block is:

$$\frac{a}{2} = \frac{4.94}{2} = 2.47 \text{ in}$$

$$d_e = h_{beam} + t_{slab} - \bar{y} - \frac{a}{2} = 72 + 9 - 4.00 - 2.47 = 74.53 \text{ in}$$

3. Calculate the horizontal shear per unit length (5.8.4.1)

The horizontal shear per unit length can be calculated using:

$$V_h = \frac{V_u}{d_e}$$

$$V_h = \frac{247.4}{74.53} = 3.32 \text{ k / in}$$

$$V_n = \frac{V_h}{\varphi} = \frac{3.32}{0.9} = 3.69 \text{ k / in}$$

4. Calculate the required transverse reinforcement spacing

The required transverse reinforcement spacing can be calculated using:

$$s = \frac{\mu\left(A_{vf} f_y + P_c\right)}{V_n - cb_v}$$

$A_{vf} = 0.40 \text{ in}^2$ (2 No. 4 bars)

For normal weight concrete placed against clean, hardened concrete with surface intentionally roughened to an amplitude of 0.25 inch:

$c = 0.100$ ksi

$\mu = 1.0\lambda = (1.0)(1.0) = 1.0$

$P_c = 0$ k

$$s = \frac{(1.0)\left[(0.40)(60) + 0\right]}{3.69 - (0.100)(42)} = -47.1 \text{ in}$$

Use 2 No. 4 bars at the 12-inch spacing used for web shear. The area of transverse reinforcement provided is:

$$A_{vf} = \frac{0.40}{12} = 0.033 \text{ in}^2 \text{ / in}$$

5. Check the upper limit on the amount of nominal shear resistance used in design (5.8.4.1)

The nominal shear resistance used in design shall be the lesser of:

$V_n \leq 0.2 f'_c A_{cv}$

$V_n \leq 0.8 A_{cv}$

$A_{cv} = 42 \text{ in}^2$

$0.2 f'_c A_{cv} = (0.2)(5)(42) = 42.0 \text{ k/in}$

$0.8 A_{cv} = (0.8)(42) = 33.6 \text{ k/in}$

The least of these two values is 33.6 k / in and the nominal resistance of the interface plane is:

$$V_n = cA_{cv} + \mu\left(A_{vf} f_y + P_c\right) = (0.100)(42) + (1.0)\left[(0.033)(60) + 0\right] = 6.18 \text{ k / in}$$

6.18 k / in < 33.6 k / in

The factored shear resistance is:

$V_r = \varphi V_n = (0.9)(6.18) = 5.56$ k / in > 3.32 k / in O.K.

6. Check the minimum reinforcement requirements (5.8.4.1)

The cross-sectional area, A_{vf}, of the reinforcement per unit length of the beam or girder should satisfy that required by the following:

$$V_n = cA_{cv} + \mu\left(A_{vf} f_y + P_c\right)$$

$$A_{vf} \geq \frac{0.05 b_v}{f_y}$$

The minimum reinforcement requirements may be waived if:

$$\frac{V_n}{A_{cv}} < 0.100 \text{ ksi}$$

For the first requirement, set V_n equal to the applied nominal horizontal shear and solve for A_{vf}.

$$3.69 = (0.100)(42) + (1.0)\left[\left(A_{vf}\right)(60) + 0\right]$$

$$A_{vf} = -0.009 \text{ in}^2 / \text{in}$$

For the second requirement:

$$\frac{0.05 b_v}{f_y} = \frac{(0.05)(42)}{60} = 0.035 \text{ in}^2 / \text{ in}$$

However:

$$\frac{V_n}{A_{cv}} = \frac{3.69}{42} = 0.088 \text{ ksi}$$

Since this is less than 0.100 ksi, the minimum reinforcement requirements may be waived and the transverse reinforcement provided is satisfactory.

Interior Beam – 0.044 Point

1. Calculate the factored vertical shear force for the Strength I Limit State, composite loads, V_u

$V_u = (1.25)(7.6) + (1.5)(12.3) + (1.75)(110.5) = 221.3$ k

2. Calculate the distance, d_e

The distance between the centroid of the steel on the tension side of the beam to the center of the compression block in the deck for the interior beam is the same as the exterior beam, same strand pattern and section properties, 74.53 inches.

3. Calculate the horizontal shear per unit length (5.8.4.1)

$$V_h = \frac{221.3}{74.53} = 2.97 \text{ k / in}$$

$$V_n = \frac{V_h}{\varphi} = \frac{2.97}{0.9} = 3.30 \text{ k / in}$$

4. Calculate the required transverse reinforcement spacing

$$s = \frac{(1.0)\left[(0.40)(60) + 0\right]}{3.30 - (0.100)(42)} = -26.7 \text{ in}$$

Use 2 No. 4 bars at the 12-inch spacing used for web shear. $A_{vf} = 0.033 \text{ in}^2/\text{in}$

5. Check the upper limit on the amount of nominal shear resistance used in design (5.8.4.1)

The upper limit on the amount of nominal shear resistance used in design and the nominal resistance of the interface plane are the same as the exterior beam. The factored shear resistance is:

$V_r = \varphi V_n = (0.9)(6.18) = 5.56 \text{ k/in} > 2.97 \text{ k/in}$ O.K.

6. Check the minimum reinforcement requirements (5.8.4.1)

For the first requirement, set V_n equal to the applied nominal horizontal shear and solve for A_{vf}.

$$3.30 = (0.100)(42) + (1.0)\left[(A_{vf})(60) + 0\right]$$

$$A_{vf} = -0.015 \text{ in}^2/\text{in}$$

For the second requirement:

$A_{vf} = 0.035 \text{ in}^2/\text{in}$

However:

$$\frac{V_n}{A_{cv}} = \frac{3.30}{42} = 0.079 \text{ ksi}$$

Since this is less than 0.100 ksi, the minimum reinforcement requirements may be waived and the transverse reinforcement provided is satisfactory.

DEFLECTIONS

The beam deflections consist of instantaneous and long-term deflections.

INSTANTANEOUS DEFLECTIONS

The deflection of both the interior and exterior beams at midspan due to the initial prestressing force can be calculated using the following, for beams with two harp points, which uses the distance to the center of gravity of the prestressing force:

$$+\Delta = \frac{PL^2}{8E_{ci}I_g}\left[e_c - (e_c - e_e)\frac{4}{3}\left(\frac{a}{L}\right)^2\right]$$

e_c = the distance from the center of gravity of the beam to the center of gravity of the prestressing steel at the midspan of the beam (in)

e_e = the distance from the center of gravity of the beam to the center of gravity of the prestressing steel at the end of the beam (in)

a = the distance from the end of the beam to the nearest harp point (in)

L = the beam length (in)

P_t = the prestressing force at transfer (k)

a = 48.5 feet = 582 inches

L = 121 feet = 1452 inches (casting length of the beam)

$$\delta = \frac{(16912)(1452)^2}{(8)(4821)(733320)}\left[32.09 + (14.38 - 32.09)\left(\frac{4}{3}\right)\frac{(582)^2}{(1452)^2}\right] = 3.57\,\text{in}\uparrow$$

w_{beam} = 1.130 k/foot = 0.0942 k/inch

The deflection at midspan due to the beam dead load using the casting length of the beam:

$$\delta_{beam} = \frac{5w_{beam}L^4}{384E_{ci}I_g} = \frac{(5)(0.0942)(1452)^4}{(384)(4821)(733320)} = 1.54\,\text{in}\downarrow$$

Exterior Beam

w_{slab} = 1.069 k/foot = 0.0891 k/inch

w_{rail} = 0.139 k/foot = 0.0116 k/inch

The decrease in deflection at midspan due to the loss of prestress may be taken as a proportion of the instantaneous deflection due to the initial prestressing force by applying a ratio of the time dependent losses, losses after transfer, to the effective prestressing stress.

Losses after transfer = Δf_{pLt} = 20.41 ksi

The change in deflection is then:

$$\delta = \left(\frac{20.41}{185.56}\right)(3.57) = 0.39\,\text{in}\downarrow$$

The deflection at midspan due to the slab dead load is:

$$\delta_{slab} = \frac{5w_{slab}L^4}{384E_{ci}I_g} = \frac{(5)(0.0891)(1440)^4}{(384)(4821)(733320)} = 1.41\,\text{in}\downarrow$$

The deflection at midspan due to the rail dead load is:

$$\delta_{rail} = \frac{5w_{rail}L^4}{384E_cI_{gc}} = \frac{(5)(0.0116)(1440)^4}{(384)(5154)(1485884)} = 0.08\,\text{in}\downarrow$$

Interior Beam

w_{slab} = 1.069 k/foot = 0.0891 k/inch

The decrease in deflection at midspan due to the loss of prestress:

The losses after transfer = 20.51 ksi

The change in deflection is:

$$\delta = \left(\frac{20.51}{185.56}\right)(3.57) = 0.39\,\text{in}\downarrow$$

The deflection at midspan due to the slab dead load is:

$$\delta_{slab} = \frac{(5)(0.0891)(1440)^4}{(384)(4821)(733320)} = 1.41\,\text{in}\downarrow$$

The deflection at midspan due to the rail dead load is:

$$\delta_{rail} = \frac{(5)(0.0116)(1440)^4}{(384)(5154)(1485884)} = 0.08\,\text{in}\downarrow$$

LONG-TERM DEFLECTIONS

Long-term or time-dependent deflections can be estimated by applying a multiplier to the instantaneous deflections. This example uses the average multipliers found in the PCI Bridge Design Manual published by the Precast Prestressed Concrete Institute.

Table 9 – Exterior Beam Deflections (in)					
Load	Initial Deflection	Erection		Final	
		Multiplier	Deflection	Multiplier	Deflection
Initial prestress	3.57 ↑	1.96	7.00 ↑	2.88	10.28 ↑
Prestress loss	0.39 ↓	1.00	0.39 ↓	2.32	0.90 ↓
Beam	1.54 ↓	1.96	3.02 ↓	2.88	4.44 ↓
Slab	1.41 ↓	1.00	1.41 ↓	2.50	3.52 ↓
Rail	0.08 ↓	1.00	0.08 ↓	2.50	0.20 ↓
Total			2.10 ↑		1.22 ↑

Table 10 – Interior Beam Deflections (in)					
Load	Initial Deflection	Erection		Final	
		Multiplier	Deflection	Multiplier	Deflection
Initial prestress	3.57 ↑	1.96	7.00 ↑	2.88	10.28 ↑
Prestress loss	0.39 ↓	1.00	0.39 ↓	2.32	0.90 ↓
Beam	1.54 ↓	1.96	3.02 ↓	2.88	4.44 ↓
Slab	1.41 ↓	1.00	1.41 ↓	2.50	3.52 ↓
Rail	0.08 ↓	1.00	0.08 ↓	2.50	0.20 ↓
Total			2.10 ↑		1.22 ↑